Lecture Notes

on

ENGINEERING MECHANICS

(Th. 4)

$1^{st}/2^{nd}$ Semester, All Branches

Prepared by

Dr. Biswajit Parida

Lecturer, Mechanical

DEPARTMENT OF MECHANICAL ENGINEERING

GOVERNMENT POLYTECHNIC, KENDRAPARA

Kendrapara 754289

ENGINEERING MEHANICS

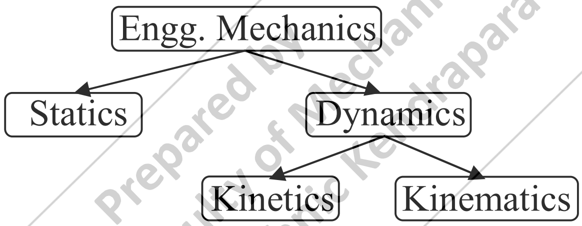
Chapter-01: Fundamentals of Engineering Mechanics

1.1 Fundamentals

Science: It may be defined as the growth of ideas through observation and experimentation.

Applied Science: The branch of science, which co-ordinates the research work, for practical utility and services of the mankind (by employing observation and experimentation), is known as Applied Science.

Engineering Mechanics: It deals with the laws and principles of Mechanics, along with their applications to engineering problems.



Statics: It deals with the forces and their effects, while acting upon the bodies at rest.

Dynamics: It deals with the forces and their effects, while acting upon the bodies in motion.

Kinetics: It deals with the bodies in motion due to the application of forces.

Kinematics: It deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

Rigid Bodies: A rigid body consists of a system of innumerable particles. If the positions of its various particles remain fixed, relative to one another (or in other words, distance between any two of its particles remain constant), it is called a solid or rigid body.

Mass: The quantity of the matter possessed by a body is called mass. The mass of a body cannot change unless the body is damaged and part of it is physically separated. It is a measure of an object's resistance to acceleration (a change in its state of motion) when a force is applied and is a property of a physical body.

Weight: The weight of an object is usually taken to be the force on the object due to gravity.

Length: Length is the most extended dimension of an object. (The measurement or extent of something from end to end; the greater of two or the greatest of three dimensions of an object). It is a concept to measure linear distances.

Time: Time is the measure of succession of events. Time is the indefinite continued progress of existence and events that occur in apparently irreversible succession from the past through the present to the future. The successive event selected is the rotation of earth about its own axis and this is called a day.

Displacement: It is defined as the distance moved by a body/particle in the specified direction.

Velocity: The rate of change of displacement with respect to time is defined as velocity.

Acceleration: It is the rate of change of velocity with respect to time.

Momentum: The product of mass and velocity is called momentum. Thus

$$\text{Momentum} = \text{Mass} \times \text{Velocity}.$$

Unit:

A unit of measurement is a definite magnitude of a quantity, defined and adopted by convention or by law that is used as a standard for measurement of the same quantity. Any other value of that quantity can be expressed as a simple multiple of the unit of measurement. For example, length is a physical quantity.

Dimension:

A measurable extent of a particular kind, such as length, breadth, depth, or height.

Fundamental Units:

Every quantity is measured in terms of some arbitrary, but internationally accepted units, called fundamental units.

All the physical quantities, met with in Engineering Mechanics, are expressed in terms of three fundamental quantities, i.e. i) Length, ii) Mass and iii) Time.

Derived Units:

Sometimes, the units are also expressed in other units (which are derived from fundamental units) known as derived units e.g. units of area, velocity, acceleration, pressure etc.

Systems of Units:

There are only four systems of units, which are commonly used and universally recognised. These are known as:

i) C.G.S. (Centimetre-Gram-Second) units,

ii) F.P.S. (Foot-Pound-Second) units,

iii) M.K.S. (Meter-Kilogram-Second) units and

iv) S.I. units (an absolute system).

S.I. Units (International System of Units):

In this system of units, the fundamental units are metre (m), kilogram (kg) and second (s) respectively.

Density (Mass density)	kg/m^3
Force	N (Newton)
Pressure	N/mm^2 or N/m^2
Work done (in joules)	J = N-m
Power in watts	W = J/s

Scalar:

The scalar quantities (or sometimes known as scalars) are those quantities which have magnitude only such as length, mass, time, distance, volume, density, temperature, speed etc.

Vector:

The vector quantities (or sometimes known as vectors) are those quantities which have both magnitude and direction such as force, displacement, velocity, acceleration, momentum etc.

a) Representation of a vector:

A vector is represented by a directed line. It may be noted that the length OA represents the magnitude of the vector $\overrightarrow{OA}$.

The direction of the vector is $\overrightarrow{OA}$ is from O (i.e., starting point) to A (i.e., end point). It is also known as vector P.

b) Unit vector:

A vector, whose magnitude is unity, is known as unit vector.

c) Equal vectors:

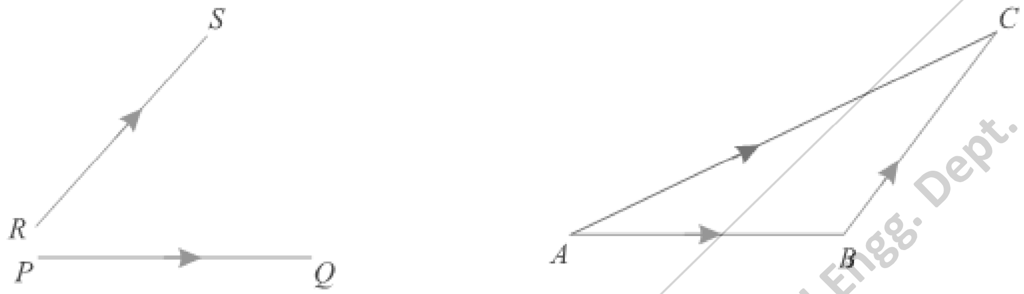
The vectors, which are parallel to each other and have same direction (i.e., same sense) and equal magnitude, are known as equal vectors.

d) Like vectors:

The vectors, which are parallel to each other and have same sense but unequal magnitude, are known as like vectors.

e) Addition of vectors:

Consider two vectors PQ and RS, which are required to be added as shown in Fig. Take a point A, and draw line AB parallel and equal in magnitude to the vector PQ to some convenient scale. Through B, draw BC parallel and equal to vector RS to the same scale. Join AC which will give the required sum of vectors PQ and RS as shown in Fig.

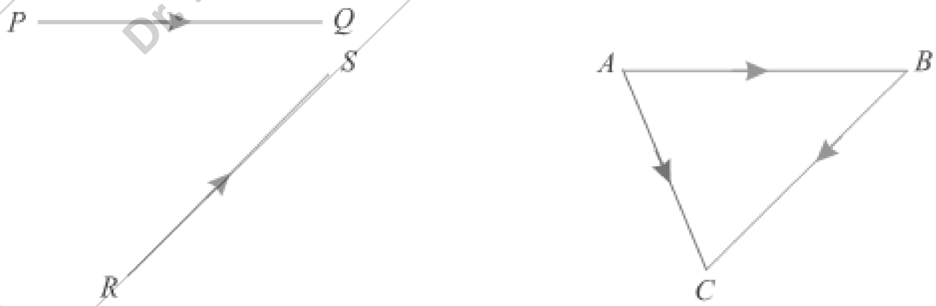


(*a*) Vectors *PQ* and *RS* (*b*) Addition of vectors

This method of adding the two vectors is called the Triangle Law of Addition of Vectors. Similarly, if more than two vectors are to be added, the same may be done first by adding the two vectors, and then by adding the third vector to the resultant of the first two and so on. This method of adding more than two vectors is called Polygon Law of Addition of Vectors.

f) Subtraction of vectors:

Consider two vectors PQ and RS in which the vector RS is required to be subtracted as shown in Fig. Take a point A, and draw line AB parallel and equal in magnitude to the vector PQ to some convenient scale. Through B, draw BC parallel and equal to the vector RS, but in opposite direction, to that of the vector RS to the same scale.



(*a*) Vectors *PQ* and *RS* (*b*) Subtraction of vectors

Join AC, which will give the resultant when the vector PQ is subtracted from vector RS as shown in Fig.

1.2 Force

It may be defined as an agent which produces or tends to produce, destroys or tends to destroy motion. Unit in SI units is N (Newton).

Effects of a Force:

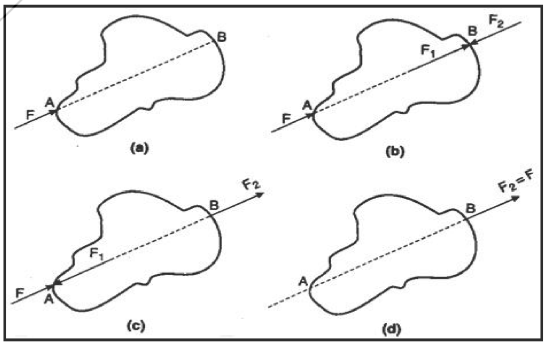
- It may change the motion of a body. i.e. if a body is at rest, the force may set it in motion and if the body is already in motion, the force may accelerate it.
- It may retard the motion of a body.
- It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.
- It may give rise to the internal stresses in the body, on which it acts.

Characteristics of a Force:

- Magnitude of the force (i.e., 100 N, 50 N, 20 kN, 5 kN, etc.)
- The direction of the line, along which the force acts (i.e., along OX, OY, at 30° North of East etc.). It is also known as line of action of the force.
- Nature of the force (i.e., whether the force is push or pull). This is denoted by placing an arrow head on the line of action of the force.
- The point at which (or through which) the force acts on the body.

Principle of Transmissibility of Forces:

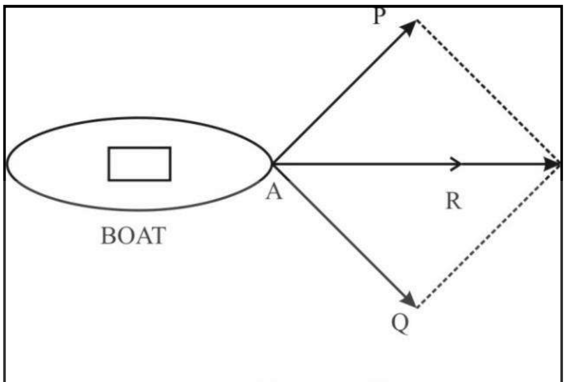
It states, "If a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body".



Principle of Superposition of Forces:

This principle states that the combined effect of force system acting on a particle or a rigid body is the sum of effects of individual forces.

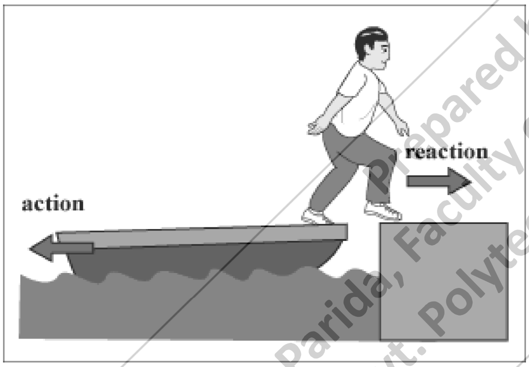
Consider two forces P and Q acting at A on a boat as shown in Fig. Let R be the resultant of these two forces P and Q. According to Newton's second law of motion, the boat will move in the direction of resultant force R with acceleration proportional to R. The same motion can be obtained when P and Q are applied simultaneously.

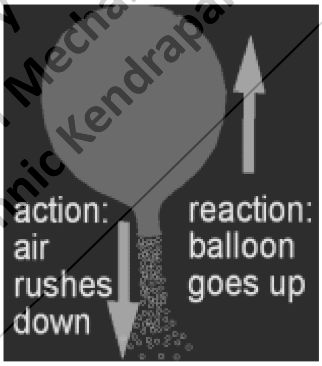


Principle of Superposition

Action & Reaction Forces:

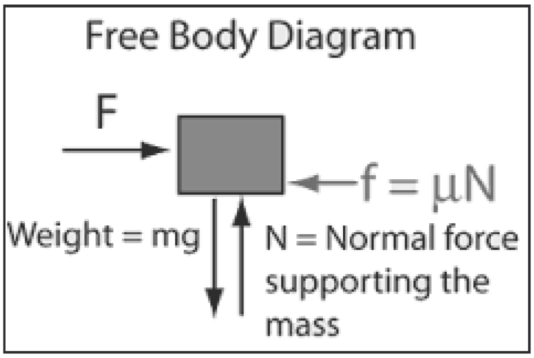
The force exerted by one of the bodies on the other is called action while the force exerted by the other on the first is called reaction. Whenever two bodies interact with each other, an action-reaction pair of force arises. Whenever an object A exerts a force on another object B, B will exert an equal and opposite force on A.





Free Body Diagram:

Free body diagrams (otherwise known as FBD's) are simplified representations in a problem of an object (the body), and the force vectors acting on it. This body is free because the diagram will show it without its surroundings; i.e. the body is 'free' of its environment.

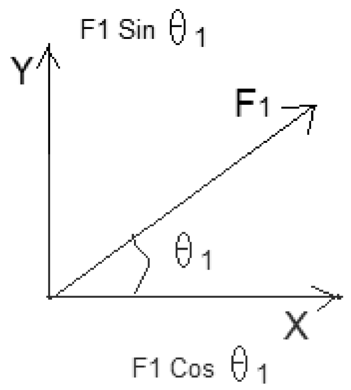


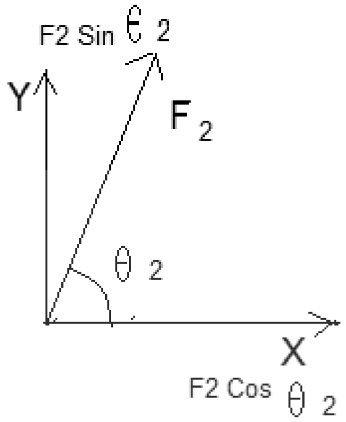
1.3 Resolution of a Force

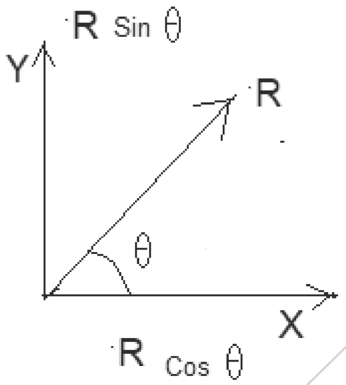
The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions.

Principle of Resolution

It states, "The algebraic sum of the resolved parts of a no. of forces, in a given direction, is equal to the resolved part of their resultant in the same direction".







$$R\cos\theta = F_1\cos\theta_1 + F_2\cos\theta_2$$

$$R\sin\theta = F_1\sin\theta_1 + F_2\sin\theta_2$$

Method of Resolution

- Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (i.e., ∑H).
- Resolve all the forces vertically and find the algebraic sum of all the vertical components (i.e., ∑V).
- The resultant R of the given forces will be given by the equation :

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

- The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan\theta = \frac{\sum V}{\sum H}$$

Notes:

- When ∑V is +ve, the resultant makes an angle between 0° and 180°. But when ∑V is -ve, the resultant makes an angle between 180° and 360°.
- When ∑H is +ve, the resultant makes an angle between 0° to 90° or 270° to 360°. But when ∑H is -ve, the resultant makes an angle between 90° to 270°.

Example 2.5. *A triangle ABC has its side AB = 40 mm along positive x-axis and side BC = 30 mm along positive y-axis. Three forces of 40 N, 50 N and 30 N act along the sides AB, BC and CA respectively. Determine magnitude of the resultant of such a system of forces.*

Solution. The system of given forces is shown in Fig. 2.3.

From the geometry of the figure, we find that the triangle *ABC* is a right angled triangle, in which the *side AC = 50 mm. Therefore

$$\sin\theta = \frac{30}{50} = 0.6$$

and $$\cos\theta = \frac{40}{50} = 0.8$$

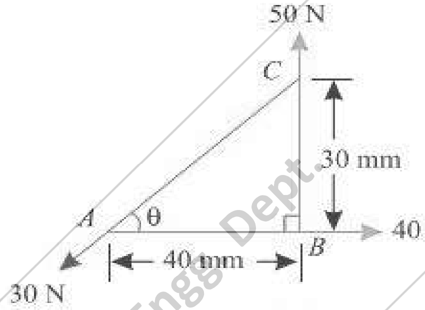


Fig. 2.3.

Resolving all the forces horizontally (*i.e.*, along *AB*),

$$\Sigma H = 40 - 30\cos\theta$$
$$= 40 - (30 \times 0.8) = 16 \text{ N}$$

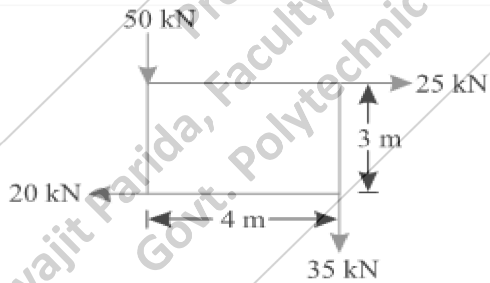
and now resolving all the forces vertically (*i.e.*, along *BC*)

$$\Sigma V = 50 - 30\sin\theta$$
$$= 50 - (30 \times 0.6) = 32 \text{ N}$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(16)^2 + (32)^2} = 35.8 \text{ N} \quad \textbf{Ans.}$$

Example 2.6. *A system of forces are acting at the corners of a rectangular block as shown in Fig. 2.4.*



Determine the magnitude and direction of the resultant force.

Solution. Given : System of forces

Magnitude of the resultant force

Resolving forces horizontally,

$$\Sigma H = 25 - 20 = 5 \text{ kN}$$

and now resolving the forces vertically

$$\Sigma V = (-50) + (-35) = -85 \text{ kN}$$

∴ Magnitude of the resultant force

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(5)^2 + (-85)^2} = 85.15 \text{ kN} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle which the resultant force makes with the horizontal.

We know that

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{-85}{5} = -17 \quad \text{or} \quad \theta = 86.6°$$

Since ΣH is positive and ΣV is negative, therefore resultant lies between 270° and 360°. Thus actual angle of the resultant force

$$= 360° - 86.6° = 273.4° \quad \textbf{Ans.}$$

Example 2.7. *The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.*

Solution. The system of given forces is shown in Fig. 2.5

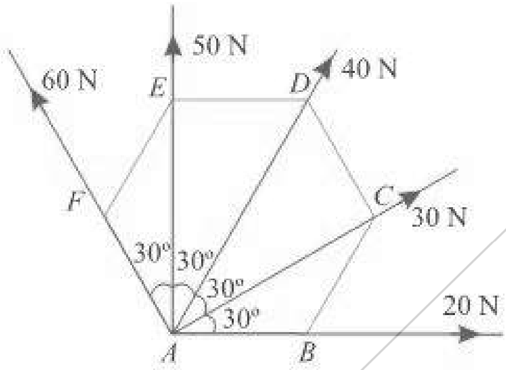


Fig. 2.5.

Magnitude of the resultant force

Resolving all the forces horizontally (*i.e.*, along *AB*),

$$\Sigma H = 20 \cos 0° + 30 \cos 30° + 40 \cos 60° + 50 \cos 90° + 60 \cos 120° \text{ N}$$

$$= (20 \times 1) + (30 \times 0.866) + (40 \times 0.5) + (50 \times 0) + 60\,(-0.5) \text{ N}$$

$$= 36.0 \text{ N} \quad \ldots(i)$$

and now resolving the all forces vertically (*i.e.*, at right angles to *AB*),

$$\Sigma V = 20 \sin 0° + 30 \sin 30° + 40 \sin 60° + 50 \sin 90° + 60 \sin 120° \text{ N}$$

$$= (20 \times 0) + (30 \times 0.5) + (40 \times 0.866) + (50 \times 1) + (60 \times 0.866) \text{ N}$$

$$= 151.6 \text{ N} \quad \ldots(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(36.0)^2 + (151.6)^2} = 155.8 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the horizontal (*i.e.*, *AB*).

We know that

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{151.6}{36.0} = 4.211 \quad \text{or} \quad \theta = 76.6° \quad \textbf{Ans.}$$

Note. Since both the values of ΣH and ΣV are positive, therefore actual angle of resultant force lies between 0° and 90°.

Example 2.8. *The following forces act at a point :*

(*i*) *20 N inclined at 30° towards North of East,*

(*ii*) *25 N towards North,*

(*iii*) *30 N towards North West, and*

(*iv*) *35 N inclined at 40° towards South of West.*

Find the magnitude and direction of the resultant force.

Solution. The system of given forces is shown in Fig. 2.6.

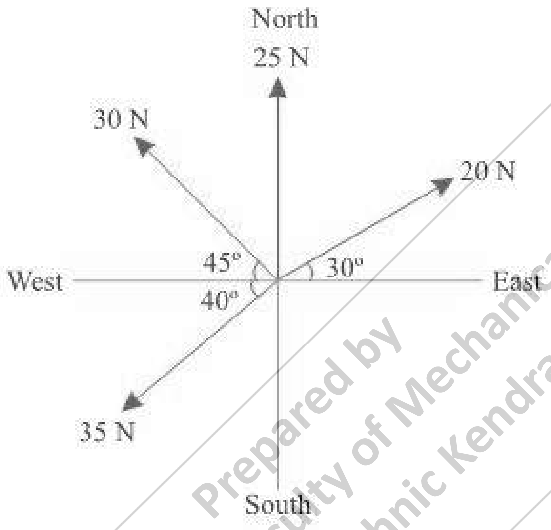


Fig. 2.6.

Magnitude of the resultant force

Resolving all the forces horizontally *i.e.*, along East-West line,

$$\Sigma H = 20 \cos 30° + 25 \cos 90° + 30 \cos 135° + 35 \cos 220° \text{ N}$$

$$= (20 \times 0.866) + (25 \times 0) + 30(-0.707) + 35(-0.766) \text{ N}$$

$$= -30.7 \text{ N} \quad \ldots(i)$$

Magnitude of the resultant force

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$$= (20 \times 0.866) + (25 \times 0) + 30(-0.707) + 35(-0.766) \text{ N}$$

$$= -30.7 \text{ N} \quad \ldots(i)$$

and now resolving all the forces vertically *i.e.*, along North-South line,

$$\Sigma V = 20 \sin 30° + 25 \sin 90° + 30 \sin 135° + 35 \sin 220° \text{ N}$$

$$= (20 \times 0.5) + (25 \times 1.0) + (30 \times 0.707) + 35\,(-0.6428)\text{ N}$$
$$= 33.7\text{ N} \qquad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(-30.7)^2 + (33.7)^2} = 45.6\text{ N} \qquad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the East.

We know that

$$\tan\theta = \frac{\sum V}{\sum H} = \frac{33.7}{-30.7} = -1.098 \quad \text{or} \quad \theta = 47.7°$$

Since $\sum H$ is negative and $\sum V$ is positive, therefore resultant lies between 90° and 180°. Thus actual angle of the resultant = 180° − 47.7° = 132.3° **Ans.**

1.5 Force System

When two or more forces act on a body, they are called to form a system of forces. A force is, generally, resolved along two mutually perpendicular directions.

a) ***Coplanar forces:*** The forces, whose lines of action lie on the same plane, are known as coplanar forces.

b) ***Collinear forces:*** The forces, whose lines of action lie on the same line, are known as collinear forces.

c) ***Concurrent forces:*** The forces, which meet at one point, are known as concurrent forces. The concurrent forces may or may not be collinear.

d) ***Coplanar concurrent forces:*** The forces, which meet at one point and their lines of action also lie on the same plane, are known as coplanar concurrent forces.

e) ***Coplanar non-concurrent forces:*** The forces, which do not meet at one point, but their lines of action lie on the same plane, are known as coplanar non-concurrent forces.

f) ***Non-coplanar concurrent forces:*** The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplanar concurrent forces.

g) ***Non-coplanar non-concurrent forces:*** The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

1.6 Composition of Forces

The process of finding out the resultant force, of a number of given forces, is called composition of forces or compounding of forces.

Resultant Force

If a number of forces, are acting simultaneously on a particle, then it is possible to find out a single force which could replace them i.e., which would produce the same effect as produced by all the given forces. This single force is called resultant force.

Laws for Resultant Force

a) Triangle Law of Forces

It states, "If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order."

b) Polygon Law of Forces

It states, "If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."

Method of Composition of Forces

1. Analytical method. **2.** Graphical method

1. Analytical Method

The resultant force, of a given system of forces, may be found out analytically by the following methods:

a) Parallelogram law of forces. **b)** Method of resolution

1. Parallelogram Law of Forces:

It states, "If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection."

Mathematically, resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos\theta}$$

and

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

where F_1 and F_2 = Forces whose resultant is required to be found out,

θ = Angle between the forces F_1 and F_2, and

α = Angle which the resultant force makes with one of the forces (say F_1).

Note. It the angle (α) which the resultant force makes with the other force F_2,

then $$\tan \alpha = \frac{F_1 \sin\theta}{F_2 + F_1 \cos\theta}$$

Cor.

1. If $\theta = 0$ *i.e.*, when the forces act along the same line, then
$$R = F_1 + F_2 \qquad \text{...(Since } \cos 0° = 1)$$
2. If $\theta = 90°$ *i.e.*, when the forces act at right angle, then
$$\theta = R = \sqrt{F_1^2 + F_2^2} \qquad \text{...(Since } \cos 90° = 0)$$
3. If $\theta = 180°$ *i.e.*, when the forces act along the same straight line but in opposite directions, then $R = F_1 - F_2$ ...(Since $\cos 180° = -1$)
In this case, the resultant force will act in the direction of the greater force.
4. If the two forces are equal *i.e.*, when $F_1 = F_2 = F$ then
$$R = \sqrt{F^2 + F^2 + 2F^2 \cos\theta} = \sqrt{2F^2(1 + \cos\theta)}$$
$$= \sqrt{2F^2 \times 2\cos^2\left(\frac{\theta}{2}\right)} \qquad \text{...}\left[\because 1 + \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)\right]$$
$$= \sqrt{4F^2 \cos^2\left(\frac{\theta}{2}\right)} = 2F\cos\left(\frac{\theta}{2}\right)$$

Example 2.2. *Two forces act at an angle of 120°. The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force.*

Solution. Given : Angle between the forces $\angle AOC = 120°$, Bigger force $(F_1) = 40$ N and angle between the resultant and F_2 $(\angle BOC) = 90°$;

Let F_2 = Smaller force in N

From the geometry of the figure, we find that $\angle AOB$,
$$\alpha = 120° - 90° = 30°$$

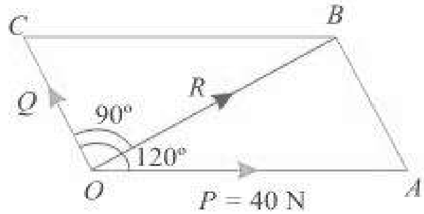


Fig. 2.1.

We know that
$$\tan \alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$
$$\tan 30° = \frac{F_2 \sin 120°}{40 + F_2 \cos 120°} = \frac{F_2 \sin 60°}{40 + F_2(-\cos 60°)}$$
$$\therefore \quad 0.577 = \frac{F_2 \times 0.866}{40 - F_2 \times 0.5} = \frac{0.866\, F_2}{40 - 0.5\, F_2}$$
$$40 - 0.5\,F_2 = \frac{0.866\,F_2}{0.577} = 1.5\,F_2$$
$$\therefore \quad 2F_2 = 40 \quad \text{or} \quad F_2 = 20 \quad \textbf{Ans.}$$

Example 2.3. *Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they Act at 60°, their resultant is $\sqrt{13}$ N.*

Solution. Given : Two forces = F_1 and F_2.

First of all, consider the two forces acting at right angles. We know that when the angle between the two given forces is 90°, then the resultant force (R)
$$\sqrt{10} = \sqrt{F_1^2 + F_2^2}$$
or $$10 = F_1^2 + F_2^2 \qquad \text{...(Squaring both sides)}$$

Similarly, when the angle between the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos 60°}$$

$\therefore$ $$13 = F_1^2 + F_2^2 + 2F_1 F_2 \times 0.5 \qquad \text{...(Squaring both sides)}$$

or $$F_1 F_2 = 13 - 10 = 3 \qquad \text{...(Substituting } F_1^2 + F_2^2 = 10)$$

We know that $(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2 = 10 + 6 = 16$

$\therefore$ $$F_1 + F_2 = \sqrt{16} = 4 \qquad \text{...}(i)$$

Similarly $$(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2 = 10 - 6 = 4$$

$\therefore$ $$F_1 - F_2 = \sqrt{4} = 2 \qquad \text{...}(ii)$$

Solving equations (i) and (ii),

$$F_1 = 3 \text{ N} \quad \text{and} \quad F_2 = 1 \text{ N} \quad \textbf{Ans.}$$

2. Graphical Method

It is another method of finding out the magnitude and direction of the resultant force by:

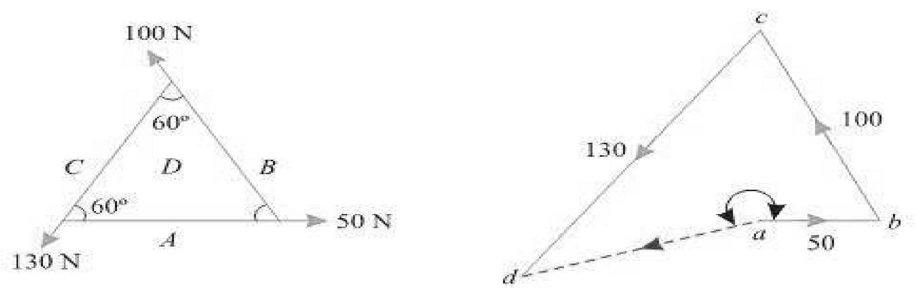
<u>**Polygon Law of Forces**</u> ("If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."):

a. **Construction of space diagram (position diagram):** It means the construction of a diagram showing the various forces (or loads) alongwith their magnitude and lines of action.
b. **Use of Bow's notations:** All the forces in the space diagram are named by using the Bow's notations. It is a convenient method in which every force (or load) is named by two capital letters, placed on its either side in the space diagram.
c. **Construction of vector diagram (force diagram):** It means the construction of a diagram starting from a convenient point and then go on adding all the forces vectorially one by one (keeping in view the directions of the forces) to some suitable scale. Now the closing side of the polygon, taken in opposite order, will give the **magnitude** of the **resultant force** (to the scale) and its direction.

Example 2.10. *A particle is acted upon by three forces equal to 50 N, 100 N and 130 N, along the three sides of an equilateral triangle, taken in order. Find graphically the magnitude and direction of the resultant force.*

Solution. The system of given forces is shown in Fig. 2.8 (a)

First of all, name the forces according to Bow's notations as shown in Fig. 2.8 (a). The 50 N force is named as AD, 100 N force as BD and 130 N force as CD.



(a) Space diagram (b) Vector diagram

Fig. 2.8.

Now draw the vector diagram for the given system of forces as shown in Fig. 2.8 (*b*) and as discussed below :

1. Select some suitable point *a* and draw *ab* equal to 50 N to some suitable scale and parallel to the 50 N force of the space diagram.
2. Through *b*, draw *bc* equal to 100 N to the scale and parallel to the 100 N force of the space diagram.
3. Similarly through *c*, draw *cd* equal to 130 N to the scale and parallel to the 130 N force of the space diagram.

Example 2.11 *The following forces act at a point :*

(*i*) *20 N inclined at 30° towards North of East.*

(*ii*) *25 N towards North.*

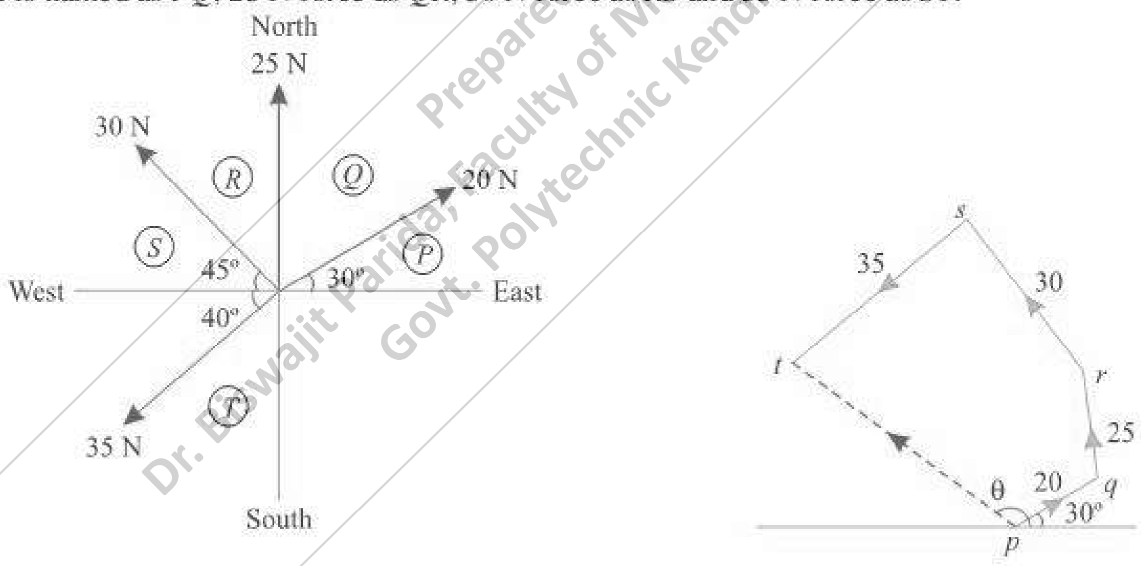
(*iii*) *30 N towards North West and*

(*iv*) *35 N inclined at 40° towards South of West.*

Find the magnitude and direction of the resultant froce.

***Solution.** The system of given forces is shown in Fig. 2.9 (*a*).

First of all, name the forces according to Bow's notations as shown in Fig. 2.9 (*a*). The 20 N force is named as *PQ*, 25 N force as *QR*, 30 N force as *RS* and 35 N force as *ST*.



(*a*) Space diagram (*b*) Vector diagram

Fig. 2.9.

Now draw the vector diagram for the given system of forces as shown in Fig. 2.9 (*b*) and as discussed below :

1. Select some suitable point *p* and draw *pq* equal to 20 N to some suitable scale and parallel to the force *PQ*.
2. Through *q*, draw *qr* equal to 25 N to the scale and parallel to the force *QR* of the space diagram.
3. Now through *r*, draw *rs* equal to 30 N to the scale and parallel to the force *RS* of the space diagram.
4. Similarly, through *s*, draw *st* equal to 35 N to the scale and parallel to the force *ST* of the space diagram.
5. Joint *pt*, which gives the magnitude as well as direction of the resultant force.
6. By measurement, we find that the magnitude of the resultant force is equal to 45.6 N and acting at an angle of 132° with the horizontal *i.e.* East–West line. **Ans.**

1.4 Moment of Force

- It is the turning effect produced by a force, on the body, on which it acts.
- The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.

Mathematically,

Moment, $M = P \times l$

where P = Force acting on the body, and

l = Perpendicular distance between the point, about which the moment is required and the line of action of the force.

Consider a force P represented, in magnitude and direction, by the line AB. Let O be a point, about which the moment of this force is required to be found out. From O, draw OC perpendicular to AB. Join OA and OB.

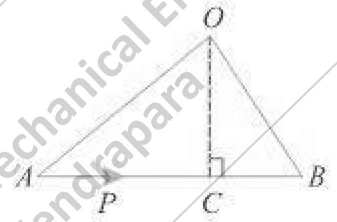


Fig. 3.1. Representation of moment

Now moment of the force P about O

$= P \times OC = AB \times OC$

But AB × OC is equal to twice the area of triangle ABO.

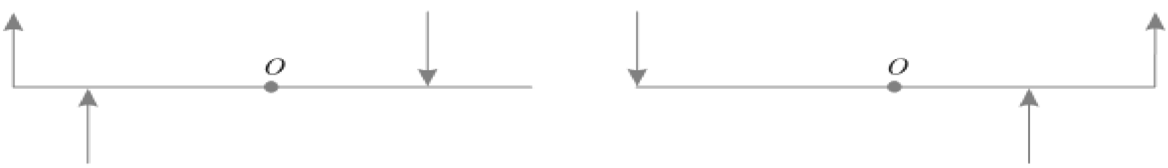
Thus the moment of a force, about any point, is equal to twice the area of the triangle, whose base is the line to some scale representing the force and whose vertex is the point about which the moment is taken.

If the force is in Newton and the distance is in meters, then the units of moment will be Newton-meter (briefly written as **N-m**). Similarly, the units of moment may be kN-m (i.e. **kN × m**), N-mm (i.e. **N × mm**) etc.

Types of Moments

a) Clockwise Moment

It is the moment of a force, whose effect is to turn or rotate the body, about the point in the same direction in which hands of a clock move.



(a) Clockwise moments (b) Anticlockwise moments

b) Anticlockwise Moment

It is the moment of a force, whose effect is to turn or rotate the body, about the point in the opposite direction in which the hands of a clock move.

Sign Convention

The general convention is to take clockwise moment as positive and anticlockwise moment as negative.

Law of Moments or Varignon's Principle of Moments

It states, "If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point."

Example 3.4. *A uniform wheel of 600 mm diameter, weighing 5 kN rests against a rigid rectangular block of 150 mm height as shown in Fig. 3.8.*

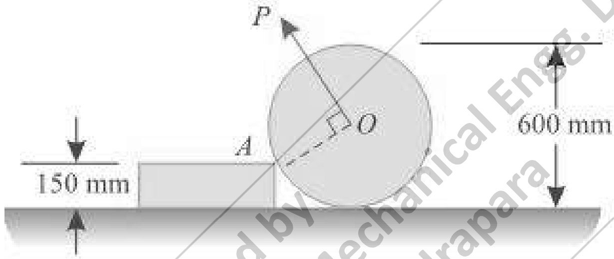


Fig. 3.8.

Find the least pull, through the centre of the wheel, required just to turn the wheel over the corner A of the block. Also find the reaction on the block. Take all the surfaces to be smooth.

Solution. Given : Diameter of wheel = 600 mm; Weight of wheel = 5 kN and height of the block = 150 mm.

Least pull required just to turn the wheel over the corner.

Let P = Least pull required just to turn the wheel in kN.

A little consideration will show that for the least pull, it must be applied normal to AO. The system of forces is shown in Fig. 3.9. From the geometry of the figure, we find that

$$\sin\theta = \frac{150}{300} = 0.5 \quad \text{or} \quad \theta = 30°$$

and

$$AB = \sqrt{(300)^2 - (150)^2} = 260 \text{ mm}$$

Now taking moments about A and equating the same,

$$P \times 300 = 5 \times 260 = 1300$$

$$\therefore \quad P = \frac{1300}{300} = 4.33 \text{ kN} \quad \textbf{Ans.}$$

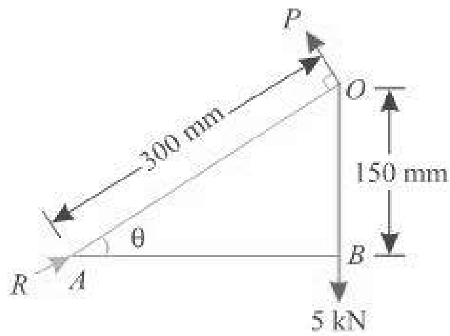


Fig. 3.9.

Reaction on the block

Let R = Reaction on the block in kN.

Resolving the forces horizontally and equating the same,

$$R\cos 30° = P\sin 30°$$

$$\therefore \quad R = \frac{P\sin 30°}{\cos 30°} = \frac{4.33 \times 0.5}{0.866} = 2.5 \text{ kN} \quad \textbf{Ans.}$$

Example 3.5. *Three forces of 2P, 3P and 4P act along the three sides of an equilateral triangle of side 100 mm taken in order. Find the magnitude and position of the resultant force.*

Solution. The system of given forces is shown in Fig. 3.11.

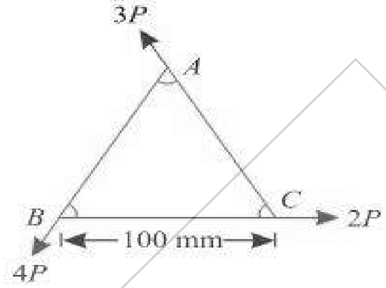


Fig. 3.11.

Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = 2P + 3P \cos 120° + 4P \cos 240°$$
$$= 2P + 3P(-0.5) + 4P(-0.5)$$
$$= -1.5\,P \qquad ...(i)$$

and now resolving all the forces vertically.

$$\Sigma V = 3P \sin 60° - 4P \sin 60°$$
$$= (3P \times 0.866) - (4P \times 0.866)$$
$$= -0.866\,P \qquad ...(ii)$$

We know that magnitude of the resultant force

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-1.5\,P)^2 + (-0.866\,P)^2} = 1.732\,P \quad \textbf{Ans.}$$

Position of the resultant force

Let x = Perpendicular distance between B and the line of action of the resultant force.

Now taking moments of the resultant force about B and equating the same,

$$1.732\,P \times x = 3P \times 100 \sin 60° = 3P \times (100 \times 0.866) = 259.8\,P$$

$$\therefore \quad x = \frac{259.8}{1.732} = 150 \text{ mm} \quad \textbf{Ans.}$$

Note. The moment of the force $2P$ and $4P$ about the point B will be zero, as they pass through it.

Example 3.6. *Four forces equal to P, 2P, 3P and 4P are respectively acting along the four sides of square ABCD taken in order. Find the magnitude, direction and position of the resultant force.*

Solution. The system of given forces is shown in Fig. 3.12.

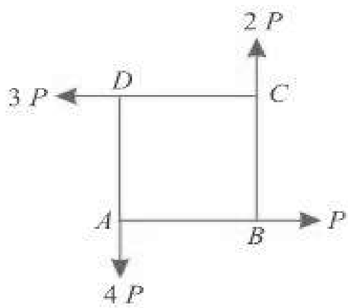


Fig. 3.12.

Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = P - 3P = -2P \qquad ...(i)$$

and now resolving all forces vertically,

$$\Sigma V = 2P - 4P = -2P \qquad ...(ii)$$

We know that magnitude of the resultant forces,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-2P)^2 + (-2P)^2}$$
$$= 2\sqrt{2}P \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant makes with the horizontal.

$$\therefore \quad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{-2P}{-2P} = 1 \quad \text{or} \quad \theta = 45°$$

Since ΣH as well as ΣV are –ve, therefore resultant lies between 180°and 270°. Thus actual angle of the resultant force = 180° + 45° = 225° **Ans.**

Position of the resultant force

Let x = Perpendicular distance between A and the line of action of the resultant force.

Now taking moments of the resultant force about A and equating the same,

$$2\sqrt{2}P \times x = (2P \times a) + (3P \times a) = 5P \times a$$

$$\therefore \quad x = \frac{5a}{2\sqrt{2}} \quad \textbf{Ans.}$$

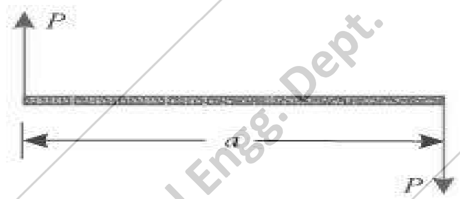
Note. The moment of the forces P and $4P$ about the point A will be zero, as they pass through it.

Couple

- A pair of two equal and unlike parallel forces (i.e. forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a couple.
- As a matter of fact, a couple is unable to produce any translatory motion (i.e., motion in a straight line).
- But it produces a motion of rotation in the body, on which it acts.

Arm of the Couple

The perpendicular distance (a), between the lines of action of the two equal and opposite parallel forces, is known as arm of the couple.



Moment of a Couple

The moment of a couple is the product of the force (i.e., one of the forces of the two equal and opposite parallel forces) and the arm of the couple. Mathematically:

Moment of a couple = P × a

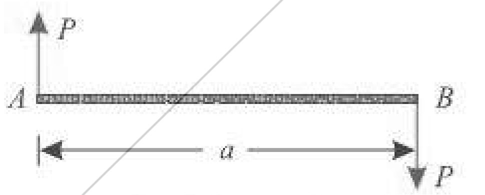
where P = Magnitude of the force, and

a = Arm of the couple.

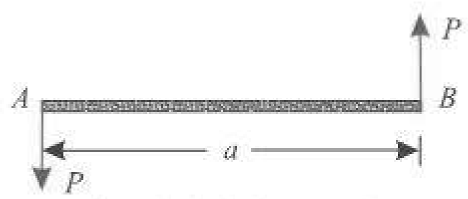
Classification of Couples

a) Clockwise Couple

A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a clockwise couple. Such a couple is also called positive couple.



(*a*) Clockwise couple



(*b*) Anticlockwise couple

b) Anticlockwise Couple

A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an anticlockwise couple. Such a couple is also called a negative couple.

Characteristics of a Couple

- The algebraic sum of the forces, constituting the couple, is zero.

- The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.
- A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.
- Any no. of coplanar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

Chapter-02: Equilibrium

2.1 Equilibrium

- If the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium.
- Such a set of forces, whose resultant is zero, are called equilibrium forces.
- The force, which brings the set of forces in equilibrium, is called an equilibrant.
- As a matter of fact, the equilibrant is equal to the resultant force in magnitude, but opposite in direction.

Principles of Equilibrium

a. ***Two force principle:*** As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.

b. ***Three force principle:*** As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.

c. ***Four force principle:*** As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

Methods for the Equilibrium of Forces

1. Analytical method. **2.** Graphical method

1. Analytical Method

2.2 Lami's Theorem:

It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the Sine of the angle between the other two." Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

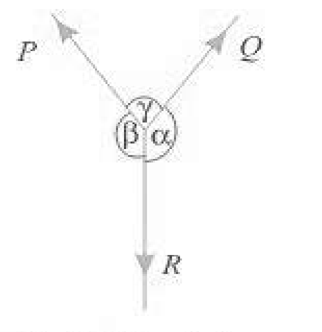


Fig. 5.1. Lami's theorem

where, P, Q, and R are three forces and α, β, γ are the angles

Proof:

Consider three coplanar forces P, Q, and R acting at a point O. Let the opposite angles to three forces be α, β and γ.

Now complete the parallelogram OACB with OA and OB as adjacent. We know that the resultant of two forces P and Q will be given by the diagonal OC both in magnitude and direction of the parallelogram OACB.

Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.

From the geometry of the figure, we find

$$BC = P \text{ and } AC = Q$$

∴ $\angle AOC = (180° - \beta)$

and $\angle ACO = \angle BOC = (180° - \alpha)$

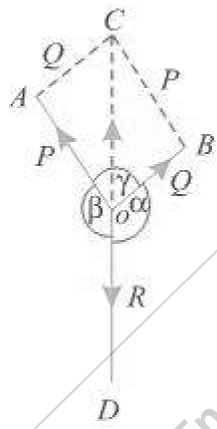


Fig. 5.2. Proof of Lami's theorem

∴ ∠ CAO = 180° – (∠ AOC + ∠ ACO)

= 180° – [(180° – β) + (180° – α)]

= 180° – 180° + β – 180° + α

= α + β – 180°

But $\alpha + \beta + \gamma = 360°$

Subtracting 180° from both sides of the above equation,

$$(\alpha + \beta - 180°) + \gamma = 360° - 180° = 180°$$

or ∠ CAO = 180° – γ

We know that in triangle AOC,

$$\frac{OA}{\sin \angle ACO} = \frac{AC}{\sin \angle AOC} = \frac{OC}{\sin \angle CAO}$$

$$\frac{OA}{\sin(180° - \alpha)} = \frac{AC}{\sin(180° - \beta)} = \frac{OC}{\sin(180° - \gamma)}$$

or $$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma} \qquad ...[\because \sin(180° - \theta) = \sin \theta]$$

Example 5.1. *An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Fig. 5.3*

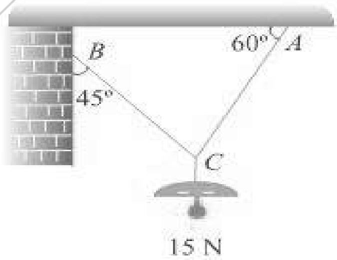


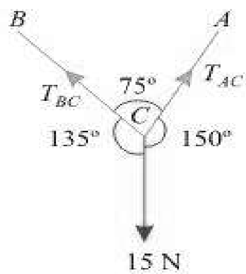
Fig. 5.3.

Using Lami's theorem, or otherwise, determine the forces in the strings AC and BC.

Solution. Given : Weight at C = 15 N

Let T_{AC} = Force in the string AC, and

T_{BC} = Force in the string BC.



The system of forces is shown in Fig. 5.4. From the geometry of the figure, we find that angle between T_{AC} and 15 N is 150° and angle between T_{BC} and 15 N is 135°.

∴ $\angle ACB = 180° - (45° + 60°) = 75°$

Applying Lami's equation at C,

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

$$\therefore \qquad T_{AC} = \frac{15 \sin 45°}{\sin 75°} = \frac{15 \times 0.707}{0.9659} = 10.98 \text{ N} \quad \textbf{Ans.}$$

and $$T_{BC} = \frac{15 \sin 30°}{\sin 75°} = \frac{15 \times 0.5}{0.9659} = 7.76 \text{ N} \quad \textbf{Ans.}$$

Example 5.3. *A light string ABCDE whose extremity A is fixed, has weights W_1 and W_2 attached to it at B and C. It passes round a small smooth peg at D carrying a weight of 300 N at the free end E as shown in Fig. 5.7.*

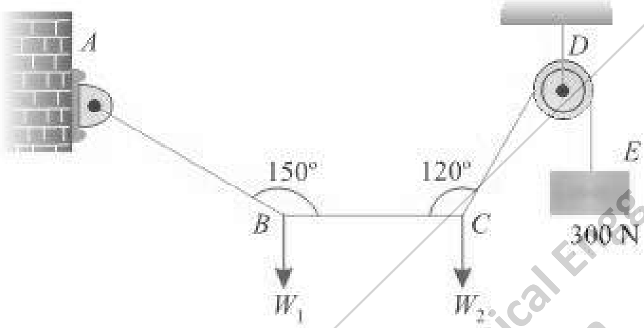
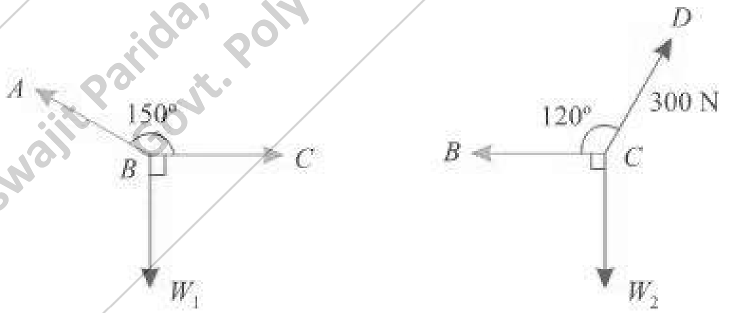


Fig. 5.7.

If in the equilibrium position, BC is horizontal and AB and CD make 150° and 120° with BC, find (i) Tensions in the portion AB, BC and CD of the string and (ii) Magnitudes of W_1 and W_2.

Solution. Given : Weight at $E = 300$ N

For the sake of convenience, let us split up the string *ABCD* into two parts. The system of forces at joints *B* and *C* is shown in Fig. 5.8. (*a*) and (*b*).



(*a*) Joint *B* (*b*) Joint *C*

Fig. 5.8.

(*i*) *Tensions is the portion AB, BC and CD of the string*

Let T_{AB} = Tension in the portion *AB*, and

T_{BC} = Tension in the portion *BC*,

We know that tension in the portion *CD* of the string.

$$T_{CD} = T_{DE} = 300 \text{ N} \quad \textbf{Ans.}$$

Applying Lami's equation at C,

$$\frac{T_{BC}}{\sin 150°} = \frac{W_2}{\sin 120°} = \frac{300}{\sin 90°}$$

$$\frac{T_{BC}}{\sin 30°} = \frac{W_2}{\sin 60°} = \frac{300}{1} \qquad \text{...}[\because \sin(180° - \theta) = \sin \theta]$$

$$\therefore \qquad T_{BC} = 300 \sin 30° = 300 \times 0.5 = 150 \text{ N} \quad \textbf{Ans.}$$

and $$W_2 = 300 \sin 60° = 300 \times 0.866 = 259.8 \text{ N}$$

and $W_2 = 300 \sin 60° = 300 \times 0.866 = 259.8$ N

Again applying Lami's equation at B,

$$\frac{T_{AB}}{\sin 90°} = \frac{W_1}{\sin 150°} = \frac{T_{BC}}{\sin 120°}$$

$$\frac{T_{AB}}{1} = \frac{W_1}{\sin 30°} = \frac{150}{\sin 60°} \qquad ...[\because \sin(180° - \theta) = \sin\theta]$$

$$\therefore \qquad T_{AB} = \frac{150}{\sin 60°} = \frac{150}{0.866} = 173.2 \text{ N} \quad \textbf{Ans.}$$

and
$$W_1 = \frac{150 \sin 30°}{\sin 60°} = \frac{150 \times 0.5}{0.866} = 86.6 \text{ N}$$

(ii) Magnitudes of W_1 and W_2

From the above calculations, we find that the magnitudes of W_1 and W_2 are 86.6 N and 259.8 N respectively. **Ans.**

2. Graphical Method

Converse of the Law of Triangle of Forces:

If three forces acting at a point be represented in magnitude and direction by the three sides a triangle, taken in order, the forces shall be in equilibrium.

Converse of the Law of Polygon of Forces:

If any number of forces acting at a point be represented in magnitude and direction by the sides of a closed polygon, taken in order, the forces shall be in equilibrium.

Example 5.11. *An electric light fixture weighing 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Fig. 5.27.*

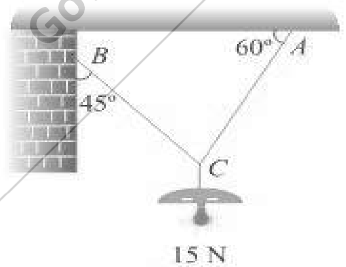


Fig. 5.27.

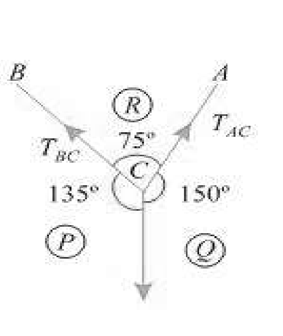
Using Lami's theorem, or otherwise, determine the forces in the strings AC and BC.

††**Solution.** Given. Weight at $C = 15$ N

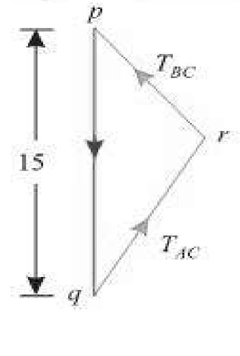
Let T_{AC} = Force in the string AC, and

T_{BC} = Force in the string BC.

First of all, draw the space diagram for the joint C and name the forces according to Bow's notations as shown in Fig. 5.28 (a). The force T_{AC} is named as RQ and the force T_{BC} as PR.



(a) Space diagram



(a) Vector diagram

Now draw the vector diagram for the given system of forces as discussed below:

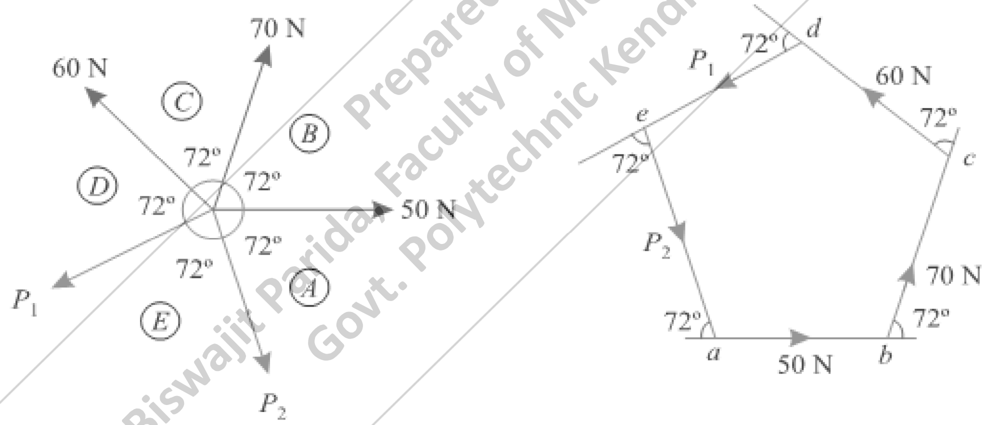
1. Select some suitable point *p* and draw a vertical line *pq* equal to 15 N to some suitable scale representing weight (*PQ*) of the electric fixture.
2. Through *p* draw a line pr parallel to PR and through q, draw a line *qr* parallel to *QR*. Let these two lines meet at r and close the triangle *pqr*, which means that joint C is in equilibrium.
3. By measurement, we find that the f4orces in strings *AC* (TAC) and *BC* (TPC) is equal to 1.0 N and 7.8 N respectively

Example 5.12. *Five strings are tied at a point and are pulled in all directions, equally spaced from one another. If the magnitude of the pulls on three consecutive strings is 50 N, 70 N and 60 N respectively, find graphically the magnitude of the pulls on two other strings.*

Solution. Given : Pulls = 50 N ; 70 N and 60 N and angle between the forces = $\frac{360}{5} = 72°$.

Let P_1 and P_2 = Pulls in the two strings.

First of all, let us draw the space diagram for the given system of forces and name them according to Bow's notations as shown in Fig 5.29 (*a*).



(*a*) Space diagram (*b*) Vector diagram

Now draw the vector diagram for the given system of forces as discussed below:

a) Select some suitable point *a* and draw a horizontal line *ab* equal to 50 N to some suitable scale representing the force AB.
b) Through *b* draw a line *bc* equal to 70 N to the scale and parallel to *BC*.
c) Similarly through *c*, draw *cd* equal to 60 N to the scale and parallel to *CD*.
d) Through *d* draw a line parallel to the force P_1 of the space diagram.
e) Similarly through *a* draw a line parallel to the force P_2 meeting the first line at *e*, thus closing the polygon *abcde*, which means that the point is in equilibrium.
f) By measurement, we find that the forces P_1 = 57.5 N and P_2 = 72.5 N respectively

Conditions of Equilibrium:

Consider a body acted upon by a number of coplanar non-concurrent forces. As a result of these forces, the body may have any one of the following states:

a) The body may move in any one direction: If the body moves in any direction, it means that there is resultant force acting on it. The body is to be at rest or in equilibrium, the resultant force causing movement must be zero. Or in other words, the horizontal component of all the forces ($\sum H$) and vertical component of all the forces ($\sum V$) must be zero. Mathematically,

$$\sum H = 0 \quad \text{and} \quad \sum V = 0.$$

b) The body may rotate about itself without moving: If the body rotates about itself, without moving, it means that there is a single resultant couple acting on it with no resultant force. The body is to be at rest or in equilibrium, the moment of the couple causing rotation must be zero. Or in other words, the resultant moment of all the forces ($\sum M$) must be zero. Mathematically,

$$\sum M = 0.$$

c) The body may move in any one direction and at the same time it may also rotate about itself: If the body moves in any direction and at the same time it rotates about itself, if means that there is a resultant force and also a resultant couple acting on it. The body is to be at rest or in equilibrium, the resultant force causing movements and the resultant moment of the couple causing rotation must be zero. Or in otherwords, horizontal component of all the forces ($\sum H$), vertical component of all the forces ($\sum V$) and resultant moment of all the forces ($\sum M$) must be zero. Mathematically,

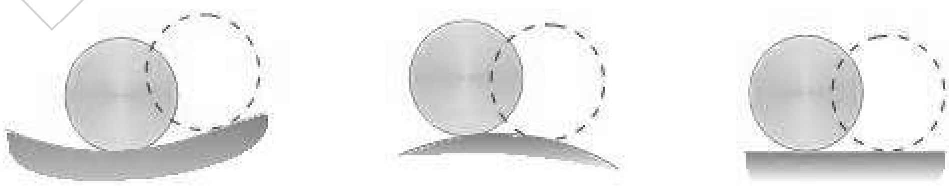
$$\sum H = 0, \quad \sum V = 0 \quad \text{and} \quad \sum M = 0$$

d) The body may be completely at rest: If the body is completely at rest, it necessarily means that there is neither a resultant force nor a couple acting on it. In this case the following conditions are already satisfied.

$$\sum H = 0 \quad \sum V = 0 \quad \text{and} \quad \sum M = 0$$

Types of Equilibrium:

From practical point of view, a body is said to be in equilibrium when it comes back to its original position, after it is slightly displaced from its position of rest. In general, following are the three types of equilibrium:



(a) Stable (b) Unstable (c) Neutral

a) Stable Equilibrium:

- A body is said to be in stable equilibrium, if it returns back to its original position, after it is slightly displaced from its position of rest.

- This happens when some additional force sets up due to displacement and brings the body back to its original position.
- A smooth cylinder, lying in a curved surface, is in stable equilibrium. If we slightly displace the cylinder from its position of rest (as shown by dotted lines), it will tend to return back to its original position in order to bring its weight normal to horizontal axis.

b) Unstable Equilibrium:

- A body is said to be in an unstable equilibrium, if it does not return back to its original position, and heels farther away, after slightly displaced from its position of rest.
- This happens when the additional force moves the body away from its position of rest. This happens when the additional force moves the body away from its position of rest.
- A smooth cylinder lying on a convex surface is in unstable equilibrium. If we slightly displace the cylinder from its position of rest (as shown by dotted lines) the body will tend to move away from its original position.

c) Neutral Equilibrium:

- A body is said to be in a neutral equilibrium, if it occupies a new position (and remains at rest in this position) after slightly displaced from its position of rest.
- This happens when no additional force sets up due to the displacement.
- A smooth cylinder lying on a horizontal plane is in neutral equilibrium.

Chapter-03: Friction

3.1 Friction

- Whenever one of the body moves or tends to move tangentially with respect to the surface, on which it rests, the interlocking property of the projecting particles opposes the motion.
- This opposing force, which acts in the opposite direction of the movement of the body, is called force of friction or simply friction.

It is of the following two types:

1. Static friction. 2. Dynamic friction.

a. ***Static Friction:*** It is the friction experienced by a body when it is at rest. Or in other words, it is the friction when the body tends to move.

b. ***Dynamic Friction:*** It is the friction experienced by a body when it is in motion. It is also called kinetic friction.

The dynamic friction is of the following two types:

i. **Sliding friction:** It is the friction, experienced by a body when it slides over another body.

ii. **Rolling friction:** It is the friction, experienced by a body when it rolls over another body.

Limiting Friction:

It has been observed that when a body, lying over another body, is gently pushed, it does not move because of the frictional force, which prevents the motion. It shows that the force of the hand is being exactly balanced by the force of friction, acting in the opposite direction. If we again push the body, a little harder, it is still found to be in equilibrium. It shows that the force of friction has increased itself so as to become equal and opposite to the applied force.

There is, however, a limit beyond which the force of friction cannot increase. If the applied force exceeds this limit, the force of friction cannot balance it and the body begins to move, in the direction of the applied force. This maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as **limiting friction**. It may be noted that when the applied force is less than the limiting friction, the body remains at rest, and the friction is called static friction, which may have any value between zero and limiting friction.

Coefficient of Friction:

It is the ratio of limiting friction to the normal reaction, between the two bodies, and is generally denoted by μ.

Mathematically, coefficient of friction,

$$\mu = \frac{F}{R} = \tan \phi \qquad \text{or} \qquad F = \mu R$$

where φ = Angle of friction,

F = Limiting friction, and

R = Normal reaction between the two bodies.

Normal Reaction:

It has been experienced that whenever a body, lying on a horizontal or an inclined surface, is in equilibrium, its weight acts vertically downwards through its centre of gravity. The surface, in turn, exerts an upward reaction on the body. This reaction, which is taken to act perpendicular to the plane, is called **normal reaction** and is, generally, denoted by R and the force of friction is directly proportional to it.

Angle of Friction:

Consider a body of weight W resting on an inclined plane as shown in Fig. 8.1. We know that the body is in equilibrium under the action of the following forces:

1. Weight (W) of the body, acting vertically downwards,

2. Friction force (F) acting upwards along the plane, and

3. Normal reaction (R) acting at right angles to the plane.

Let the angle of inclination (α) be gradually increased, till the body just starts sliding down the plane. This angle of inclined plane, at which a body just begins to slide down the plane, is called the **angle of friction**. This is also equal to the angle, which the normal reaction makes with the vertical.

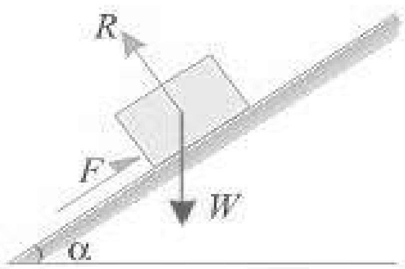


Fig. 8.1. Angle of friction.

Angle of Repose:

- The steepest angle at which a sloping surface formed of loose material is stable.
- The angle of repose, or critical angle of repose, of a granular material is the steepest angle of descent or dip relative to the horizontal plane to which a material can be piled without slumping.
- At this angle, the material on the slope face is on the verge of sliding.
- The angle of repose can range from 0° to 90°.

Laws of Friction:

Prof. Coulomb, after extensive experiments, gave some laws of friction, which may be grouped under the following heads:

1. Laws of Static Friction

Following are the laws of static friction:

a) The force of friction always acts in a direction, opposite to that in which the body tends to move, if the force of friction would have been absent.

b) The magnitude of the force of friction is exactly equal to the force, which tends to move the body.

c) The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces. Mathematically:

$$F/R = \text{Constant}$$

where F = Limiting friction, and

R = Normal reaction.

d) The force of friction is independent of the area of contact between the two surfaces.

e) The force of friction depends upon the roughness of the surfaces.

2. Laws of Kinetic or Dynamic Friction

Following are the laws of kinetic or dynamic friction:

a) The force of friction always acts in a direction, opposite to that in which the body is moving.

b) The magnitude of kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.

c) For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

Example 8.1. *A body of weight 300 N is lying on a rough horizontal plane having a coefficient of friction as 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.*

Solution. Given: Weight of the body (W) = 300 N; Coefficient of friction (μ) = 0.3 and angle made by the force with the horizontal (α) = 25°

Let P = Magnitude of the force, which can move the body, and

F = Force of friction.

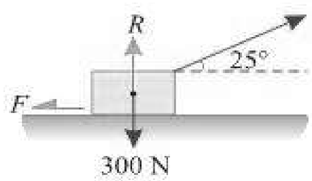


Fig. 8.2.

Resolving the forces horizontally,

$$F = P\cos\alpha = P\cos 25° = P \times 0.9063$$

and now resolving the forces vertically,

$$R = W - P\sin\alpha = 300 - P\sin 25°$$
$$= 300 - P \times 0.4226$$

We know that the force of friction (F),

$$0.9063\,P = \mu R = 0.3 \times (300 - 0.4226\,P) = 90 - 0.1268\,P$$

or

$$90 = 0.9063\,P + 0.1268\,P = 1.0331\,P$$

$$\therefore \quad P = \frac{90}{1.0331} = 87.1 \text{ N} \quad \textbf{Ans.}$$

3.2 Equilibrium of Bodies on Level Plane

Equilibrium of a Body on a Rough Horizontal Plane:

We know that a body, lying on a rough horizontal plane will remain in equilibrium. But whenever a force is applied on it, the body will tend to move in the direction of the force. In such cases, equilibrium of the body is studied first by resolving the forces horizontally and then vertically.

Now the value of the force of friction is obtained from the relation:

$$F = \mu R$$

Where μ = Coefficient of friction, and

R = Normal reaction

Equilibrium of a Body on a Rough Inclined Plane Subjected to a Force Acting along the Inclined Plane:

Consider a body lying on a rough inclined plane subjected force acting along the inclined plane, which keeps it in equilibrium as shown in Fig. 8.8. (a) and (b).

Let W = Weight of the body,

α = Angle, which the inclined plane makes with the horizontal,

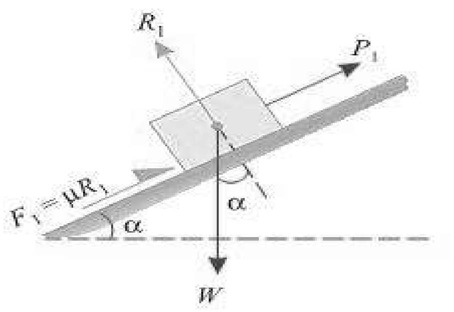
R = Normal reaction,

μ = Coefficient of friction between the body and the inclined plane, and

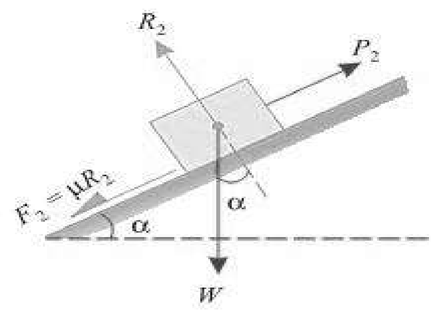
φ = Angle of friction, such that $\mu = \tan \varphi$.

A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the following two cases:

1. Minimum force (P_1) which will keep the body in equilibrium, when it is at the point of sliding downwards



(a) Body at the point of sliding downwards



(b) Body at the point of sliding upwards

In this case, the force of friction ($F_1 = \mu.R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.8 (a). Now resolving the forces along the plane,

$$P_1 = W \sin \alpha - \mu.R_1 \qquad ...(i)$$

and now resolving the forces perpendicular to the plane.

$$R_1 = W \cos \alpha \qquad ...(ii)$$

Substituting the value of R_1 in equation (i),

$$P_1 = W \sin \alpha - \mu \, W \cos \alpha = W (\sin \alpha - \mu \cos \alpha)$$

and now substituting the value of $\mu = \tan \varphi$ in the above equation,

$$P_1 = W (\sin \alpha - \tan \varphi \cos \alpha)$$

Multiplying both sides of this equation by $\cos \varphi$,

$$P_1 \cos \varphi = W (\sin \alpha \cos \varphi - \sin \varphi \cos \alpha) = W \sin (\alpha - \varphi)$$

$$\therefore \qquad P_1 = W \times \frac{\sin (\alpha - \phi)}{\cos \phi}$$

2. Maximum force (P_2) which will keep the body in equilibrium, when it is at the point of sliding upwards

In this case, the force of friction ($F_2 = \mu.R_2$) will act downwards as the body is at the point of sliding upwards as shown in Fig. 8.8 (b). Now resolving the forces along the plane,

$$P_2 = W \sin \alpha + \mu.R_2 \qquad ...(i)$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W \cos \alpha \qquad ...(ii)$$

Substituting the value of R_2 in equation (i),

$$P_2 = W \sin \alpha + \mu \, W \cos \alpha = W (\sin \alpha + \mu \cos \alpha)$$

and now substituting the value of $\mu = \tan \varphi$ in the above equation,

$$P_2 = W (\sin \alpha + \tan \varphi \cos \alpha)$$

Multiplying both sides of this equation by $\cos \varphi$,

$$P_2 \cos \varphi = W (\sin \alpha \cos \varphi + \sin \varphi \cos \alpha) = W \sin (\alpha + \varphi)$$

$$\therefore \qquad P_2 = W \times \frac{\sin (\alpha + \phi)}{\cos \phi}$$

Example 8.5. *A body of weight 500 N is lying on a rough plane inclined at an angle of 25° with the horizontal. It is supported by an effort (P) parallel to the plane as shown in Fig. 8.9.*

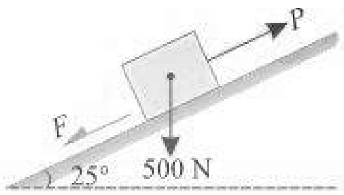


Fig. 8.9.

Determine the minimum and maximum values of P, for which the equilibrium can exist, if the angle of friction is 20°.

Solution. Given: Weight of the body (W) = 500 N ; Angle at which plane is inclined (α) = 25° and angle of friction (ϕ) = 20°.

Minimum value of P

We know that for the minimum value of P, the body is at the point of sliding downwards. We also know that when the body is at the point of sliding downwards, then the force

$$P_1 = W \times \frac{\sin(\alpha - \phi)}{\cos \phi} = 500 \times \frac{\sin(25° - 20°)}{\cos 20°} \text{ N}$$

$$= 500 \times \frac{\sin 5°}{\cos 20°} = 500 \times \frac{0.0872}{0.9397} = 46.4 \text{ N} \qquad \textbf{Ans.}$$

Maximum value of P

We know that for the maximum value of P, the body is at the point of sliding upwards. We also know that when the body is at the point of sliding upwards, then the force

$$P_2 = W \times \frac{\sin(\alpha + \phi)}{\cos \phi} = 500 \times \frac{\sin(25° + 20°)}{\cos 20°} \text{ N}$$

$$= 500 \times \frac{\sin 45°}{\cos 20°} = 500 \times \frac{0.7071}{0.9397} = 376.2 \text{ N} \qquad \textbf{Ans.}$$

Equilibrium of a Body on a Rough Inclined Plane Subjected to a Force acting Horizontally:

Consider a body lying on a rough inclined plane subjected to a force acting horizontally, which keeps it in equilibrium as shown in Fig. 8.13. (a) and (b).

W = Weight of the body,

α = Angle, which the inclined plane makes with the horizontal,

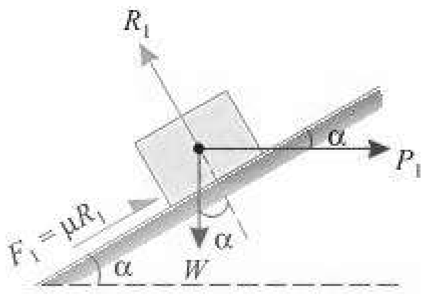
R = Normal reaction,

μ = Coefficient of friction between the body and the inclined plane, and

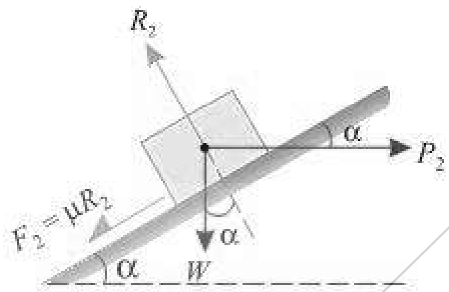
φ = Angle of friction, such that $\mu = \tan \varphi$.

A little consideration will show that if the force is not there, the body will slide down on the plane. Now we shall discuss the following two cases:

1. ***Maximum force (P_2) which will keep the body in equilibrium, when it is at the point of sliding downwards***



(a) Body at the point of sliding downwards



(b) Body at the point of sliding upwards

In this case, the force of friction ($F_1 = \mu . R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.13. (a). Now resolving the forces along the plane,

$$P_1 \cos\alpha = W \sin\alpha - \mu R_1 \qquad ...(i)$$

and now resolving the forces perpendicular to the plane,

$$R_1 = W \cos\alpha + P_1 \sin\alpha \qquad ...(ii)$$

Substituting this value of R_1 in equation (i),

$$P_1 \cos\alpha = W \sin\alpha - \mu (W \cos\alpha + P1 \sin\alpha)$$

$$= W \sin\alpha - \mu W \cos\alpha - \mu P_1 \sin\alpha$$

$$P_1 \cos\alpha + \mu P_1 \sin\alpha = W \sin\alpha - \mu W \cos\alpha$$

$$P_1 (\cos\alpha + \mu \sin\alpha) = W (\sin\alpha - \mu \cos\alpha)$$

$$\therefore \qquad P_1 = W \times \frac{(\sin\alpha - \mu\cos\alpha)}{(\cos\alpha + \mu\sin\alpha)}$$

Now substituting the value of $\mu = \tan\phi$ in the above equation,

$$P_1 = W \times \frac{(\sin\alpha - \tan\phi\cos\alpha)}{(\cos\alpha + \tan\phi\sin\alpha)}$$

Multiplying the numerator and denominator by $\cos\phi$,

$$P_1 = W \times \frac{\sin\alpha\cos\phi - \sin\phi\cos\alpha}{\cos\alpha\cos\phi + \sin\alpha\sin\phi} = W \times \frac{\sin(\alpha - \phi)}{\cos(\alpha - \phi)}$$

$$= W \tan(\alpha - \phi) \qquad ...(\text{when } \alpha > \phi)$$

$$= W \tan(\phi - \alpha) \qquad ...(\text{when } \phi > \alpha)$$

2. Maximum force (P_2) which will keep the body in equilibrium, when it is at the point of sliding upwards

In this case, the force of friction ($F_2 = \mu R_2$) will act downwards, as the body is at the point of sliding upwards as shown in Fig.8.12. (b). Now resolving the forces along the plane,

$$P_2 \cos\alpha = W \sin\alpha + \mu R_2 \qquad ...(iii)$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W \cos \alpha + P_2 \sin \alpha \quad ...(iv)$$

Substituting this value of R_2 in the equation (iii),

$$P_2 \cos \alpha = W \sin \alpha + \mu (W \cos \alpha + P_2 \sin \alpha)$$

$$= W \sin \alpha + \mu W \cos \alpha + \mu P_2 \sin \alpha$$

$$P_2 \cos \alpha - \mu P_2 \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

$$\therefore \quad P_2 = W \times \frac{(\sin \alpha + \mu \cos \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

Now substituting the value of $\mu = \tan \phi$ in the above equation,

$$P_2 = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by $\cos \phi$,

$$P_2 = W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \phi \sin \alpha} = W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)}$$

$$= W \tan (\alpha + \phi)$$

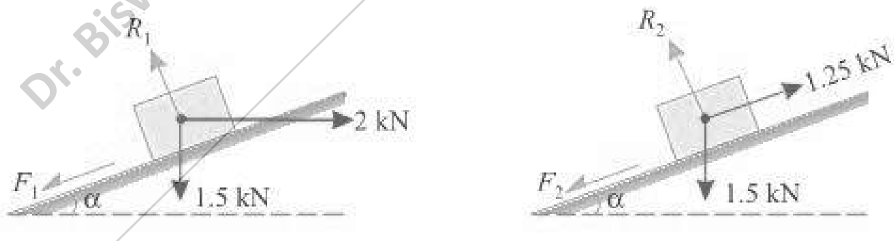
Example 8.9. *A load of 1.5 kN, resting on an inclined rough plane, can be moved up the plane by a force of 2 kN applied horizontally or by a force 1.25 kN applied parallel to the plane. Find the inclination of the plane and the coefficient of friction.*

Solution. Given: Load $(W) = 1.5$ kN; Horizontal effort $(P_1) = 2$ kN and effort parallel to the inclined plane $(P_2) = 1.25$ kN.

Inclination of the plane

Let α = Inclination of the plane, and

ϕ = Angle of friction.



(*a*) Horizontal force (*b*) Force parallel to the plane

Fig. 8.14.

First of all, consider the load of 1.5 kN subjected to a horizontal force of 2 kN as shown in Fig. 8.14 (*a*). We know that when the force is applied horizontally, then the magnitude of the force, which can move the load up the plane.

$$P = W \tan (\alpha + \phi)$$

or $$2 = 1.5 \tan (\alpha + \phi)$$

$$\therefore \quad \tan (\alpha + \phi) = \frac{2}{1.5} = 1.333 \quad \text{or} \quad (\alpha + \phi) = 53.1°$$

Now consider the load of 1.5 kN subjected to a force of 1.25 kN along the plane as shown in Fig. 8.14 (*b*). We Know that when the force is applied parallel to the plane, then the magnitude of the force, which can move the load up the plane,

$$P = W \times \frac{\sin(\alpha + \phi)}{\cos \phi}$$

or $$1.25 = 1.5 \times \frac{\sin 53.1°}{\cos \phi} = 1.5 \times \frac{0.8}{\cos \phi} = \frac{1.2}{\cos \phi}$$

∴ $$\cos \phi = \frac{1.2}{1.25} = 0.96 \quad \text{or} \quad \phi = 16.3°$$

and $$\alpha = 53.1° - 16.3° = 36.8° \quad \textbf{Ans.}$$

Coefficient of friction

We know that the coefficient of friction,

$$\mu = \tan \phi = \tan 16.3° = 0.292 \quad \textbf{Ans.}$$

∴ $$\tan(\alpha + \phi) = \frac{2}{1.5} = 1.333 \quad \text{or} \quad (\alpha + \phi) = 53.1°$$

Now consider the load of 1.5 kN subjected to a force of 1.25 kN along the plane as shown in Fig. 8.14 (*b*). We Know that when the force is applied parallel to the plane, then the magnitude of the force, which can move the load up the plane,

$$P = W \times \frac{\sin(\alpha + \phi)}{\cos \phi}$$

or $$1.25 = 1.5 \times \frac{\sin 53.1°}{\cos \phi} = 1.5 \times \frac{0.8}{\cos \phi} = \frac{1.2}{\cos \phi}$$

∴ $$\cos \phi = \frac{1.2}{1.25} = 0.96 \quad \text{or} \quad \phi = 16.3°$$

and $$\alpha = 53.1° - 16.3° = 36.8° \quad \textbf{Ans.}$$

Coefficient of friction

We know that the coefficient of friction,

$$\mu = \tan \phi = \tan 16.3° = 0.292 \quad \textbf{Ans.}$$

Equilibrium of a Body on a Rough Inclined Plane Subjected to a Force Acting at some Angle with the Inclined Plane:

Consider a body lying on a rough inclined plane subjected to a force acting at some angle with the inclined plane, which keeps it in equilibrium as shown in Fig. 8.17 (a) and (b).

Let W = Weight of the body,

α = Angle which the inclined plane makes with the horizontal,

θ = Angle which the force makes with the inclined surface,

R = Normal reaction,

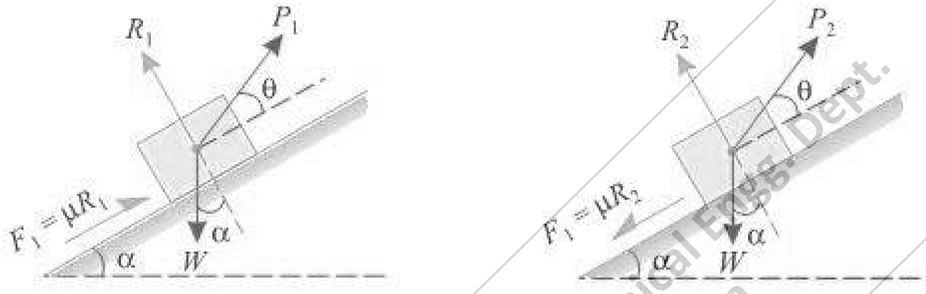
μ = Coefficient of friction between the body and the inclined plane, and

φ = Angle of friction, such that μ = tan φ.

A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the following two cases:

1. Maximum force (P_2) which will keep the body in equilibrium, when it is at the point of sliding upwards

In this case, the force of friction ($F_2 = \mu R_2$) will act downwards, as the body is at the point of sliding upwards as shown in Fig.8.12. (b). Now resolving the forces along the plane



(*a*) Body at the point of sliding downwards

(*b*) Body at the point of sliding upwards

In this case, the force of friction ($F_1 = \mu R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.17 (a). Now resolving the forces along the plane,

$$P_1 \cos\theta = W \sin\alpha - \mu R_1 \qquad \text{...(i)}$$

and now resolving the forces perpendicular to the plane,

$$R_1 = W \cos\alpha - P_1 \sin\theta \qquad \text{...(ii)}$$

Substituting the value of R_1 in equation (i),

$$P_1 \cos\theta = W \sin\alpha - \mu (W \cos\alpha - P_1 \sin\theta)$$

$$= W \sin\alpha - \mu W \cos\alpha + \mu P_1 \sin\theta$$

$$P_1 \cos\theta - \mu P_1 \sin\theta = W \sin\alpha - \mu W \cos\alpha$$

$$P_1(\cos\theta - \mu \sin\theta) = W (\sin\alpha - \mu \cos\alpha)$$

$$\therefore \quad P_1 = W \times \frac{(\sin\alpha - \mu\cos\alpha)}{(\cos\theta - \mu\sin\theta)}$$

and now substituting the value of $\mu = \tan\phi$ in the above equation,

$$P_1 = W \times \frac{(\sin\alpha - \tan\phi\cos\alpha)}{(\cos\theta - \tan\phi\sin\theta)}$$

Multiplying the numerator and denominator by $\cos\phi$,

$$P_1 = W \times \frac{(\sin\alpha\cos\phi - \sin\phi\cos\alpha)}{(\cos\theta\cos\phi - \sin\phi\sin\theta)} = W \times \frac{\sin(\alpha - \phi)}{\cos(\theta + \phi)}$$

2. Maximum force (P_2) which will keep the body in equilibrium, when it is at the point of sliding upwards

In this case, the force of friction ($F_2 = \mu R_2$) will act downwards as the body is at the point of sliding upwards as shown in Fig. 8.17 (b). Now resolving the forces along the plane.

$$P_2 \cos\theta = W \sin\alpha + \mu R_2 \quad \text{...(iii)}$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W \cos\alpha - P_2 \sin\theta \quad \text{...(iv)}$$

Substituting the value of R_2 in equation (iii),

$$P_2 \cos\theta = W \sin\alpha + \mu (W \cos\alpha - P_2 \sin\theta)$$

$$= W \sin\alpha + \mu W \cos\alpha - \mu P_2 \sin\theta$$

$$P_2 \cos\theta + \mu P_2 \sin\theta = W \sin\alpha + \mu W \cos\alpha$$

$$P_2 (\cos\theta + \mu \sin\theta) = W (\sin\alpha + \mu \cos\alpha),$$

$$\therefore \quad P_2 = W \times \frac{(\sin\alpha + \mu \cos\alpha)}{(\cos\theta + \mu \sin\theta)}$$

and now substituting the vaue of $\mu = \tan\phi$ in the above equation,

$$P_2 = W \times \frac{(\sin\alpha + \tan\phi \cos\alpha)}{(\cos\theta + \tan\phi \sin\theta)}$$

Multiplying the numerator and denominator by $\cos\phi$,

$$P_2 = W \times \frac{(\sin\alpha \cos\phi + \sin\phi \cos\alpha)}{(\cos\theta \cos\phi + \sin\phi \sin\theta)} = W \times \frac{\sin(\alpha + \phi)}{\cos(\theta - \phi)}$$

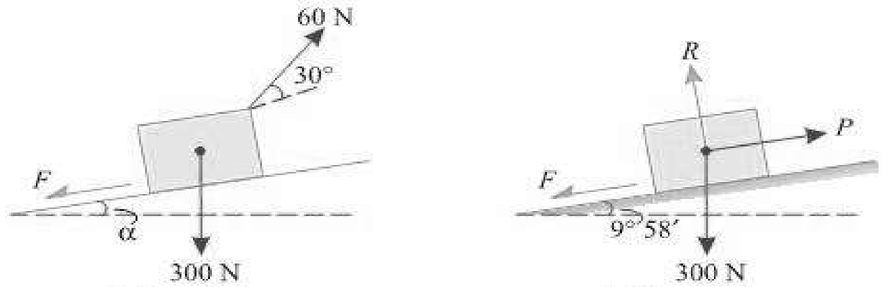
Example 8.11. *Find the force required to move a load of 300 N up a rough plane, the force being applied parallel to the plane. The inclination of the plane is such that when the same load is kept on a perfectly smooth plane inclined at the same angle, a force of 60 N applied at an inclination of 30° to the plane, keeps the same load in equilibrium.*

Assume coefficient of friction between the rough plane and the load to be equal to 0.3.

Solution. Given: Load (W) = 300 N; Force (P_1) = 60 N and angle at which force is inclined (θ) = 30°.

Let α = Angle of inclination of the plane.

First of all, consider the load lying on a smooth plane inclined at an angle (α) with the horizontal and subjected to a force of 60 N acting at an angle 30° with the plane as shown in Fig. 8.18 (*a*).



(*a*) Smooth plane (*b*) Rough plane

Fig. 8.18.

We know that in this case, because of the smooth plane $\mu = 0$ or $\phi = 0$. We also know that the force required, when the load is at the point of sliding upwards (P),

$$60 = W \times \frac{\sin(\alpha + \phi)}{\cos(\theta - \phi)} = 300 \times \frac{\sin \alpha}{\cos 30^\circ} = 300 \times \frac{\sin \alpha}{0.866} = 346.4 \sin \alpha$$

$$...(\because \phi = 0)$$

or $$\sin \alpha = \frac{60}{346.4} = 0.1732 \qquad \text{or} \qquad \alpha = 10^\circ$$

Now consider the load lying on the rough plane inclined at an angle of 10° with the horizontal as shown in Fig. 8.18. (*b*). We know that in this case, $\mu = 0.3 = \tan \phi$ or $\phi = 16.7°$.

We also know that force required to move the load up the plane,

$$P = W \times \frac{\sin(\alpha + \phi)}{\cos \phi} = 300 \times \frac{\sin (10^\circ + 16.7^\circ)}{\cos 16.7^\circ} \text{ N}$$

$$= 300 \times \frac{\sin 26.7^\circ}{\cos 16.7^\circ} = 300 \times \frac{0.4493}{0.9578} = 140.7 \text{ N} \qquad \textbf{Ans.}$$

Alternative method

Ist case

Given: In this case load (P) = 60 N; Angle (θ) = 30° and force of friction $F = 0$ (because of smooth plane). Resolving the forces along the inclined plane,

$$60 \cos 30^\circ = 300 \sin \alpha$$

$$\therefore \qquad \sin \alpha = \frac{60 \cos 30^\circ}{300} = \frac{60 \times 0.866}{300} = 0.1732 \qquad \text{or} \qquad \alpha = 10^\circ$$

2nd case

Given: In this case, coefficient of friction (μ) = 0.3 = tan ϕ or ϕ = 16.7°

Let P = Force required to move the load up the plane,

R = Normal reaction, and

F = Force of friction equal to 0.3 R.

Resolving the forces along the plane,

$$P = F + 300 \sin 10^\circ = 0.3 R + (300 \times 0.1732) = 0.3 R + 51.96 \text{ N} \qquad ...(i)$$

and now resolving the forces at right angles to the plane,

$$R = 300 \cos 10^\circ = 300 \times 0.9849 = 295.5 \text{ N} \qquad ...(ii)$$

Substituting the value of R in equation (i),

$$P = (0.3 \times 295.5) + 51.96 = 140.7 \text{ N} \qquad \textbf{Ans.}$$

Example 8.3. *Two blocks A and B of weights 1 kN and 2 kN respectively are in equilibrium position as shown in Fig. 8.4.*

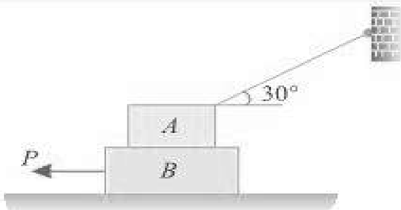
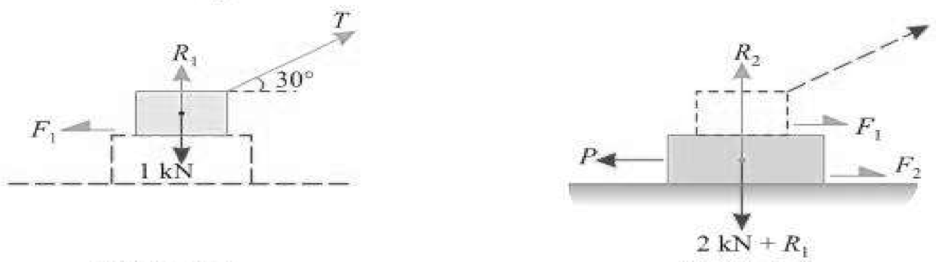


Fig. 8.4.

If the coefficient of friction between the two blocks as well as the block B and the floor is 0.3, find the force (P) required to move the block B.

Solution. Given: Weight of block A (W_A) = 1 kN; Weight of block B (W_B) = 2 kN and coefficient of friction (μ) = 0.3.



(*a*) Block *A* (*b*) Block *B*

Fig. 8.5.

The forces acting on the two blocks A and B are shown in Fig. 8.5 (a) and (b) respectively. First of all, consider the forms acitng in the block A.

Resolving the forces vertically,

$$R_1 + T \sin 30° = 1 \text{ kN}$$

or $$T \sin 30° = 1 - R_1 \quad ...(i)$$

and now resolving the forces horizontally,

$$T \cos 30° = F_1 = \mu R_1 = 0.3 R_1 \quad ...(ii)$$

Dividing equation (i) by (ii)

$$\frac{T \sin 30°}{T \cos 30°} = \frac{1 - R_1}{0.3 R_1} \quad \text{or} \quad \tan 30° = \frac{1 - R_1}{0.3 R_1}$$

$$\therefore \quad 0.5774 = \frac{1 - R_1}{0.3 R_1} \quad \text{or} \quad 0.5774 \times 0.3 \text{ R}_1 = 1 - \text{R}_1$$

or $$0.173 R_1 = 1 - R_1 \quad \text{or} \quad 1.173 R_1 = 1$$

or $$R_1 = \frac{1}{1.173} = 0.85 \text{ kN}$$

and $$F_1 = \mu . R_1 = 0.3 \times 0.85 = 0.255 \text{ kN} \quad ...(iii)$$

Now consider the block B. A little consideration will show that the downward force of the block A (equal to R_1) will also act alongwith the weight of the block B.

Resolving the forces vertically,

$$R_2 = 2 + R_1 = 2 + 0.85 = 2.85 \text{ kN}$$

$$\therefore \quad F_2 = \mu R_2 = 0.3 \times 2.85 = 0.855 \text{ kN} \quad ...(iv)$$

and now resolving the forces horizontally,

$$P = F_1 + F_2 = 0.255 + 0.855 = 1.11 \text{ kN} \quad \textbf{Ans.}$$

3.3 Ladder

Ladder Friction

The ladder is a device for climbing or scaling on the roofs or walls. It consists of two long uprights of wood, iron or rope connected by a number of cross pieces called rungs. These running serve as steps. Consider a ladder AB resting on the rough ground and leaning against a wall, as shown in Fig. 9.1.

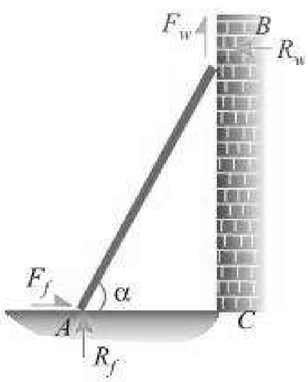


Fig. 9.1. Ladder friction

As the upper end of the ladder tends to slip downwards, therefore the direction of the force of friction between the ladder and the wall (F_w) will be upwards as shown in the figure. Similarly, as the lower end of the ladder tends to slip away from the wall, therefore the direction of the force of friction between the ladder and the floor (F_f) will be towards the wall as shown in the figure.

Since the system is in equilibrium, therefore the algebraic sum of the horizontal and vertical components of the forces must also be equal to zero.

Note: The normal reaction at the floor (R_f) will act perpendicular of the floor. Similarly, normal reaction of the wall (R_w) will also act perpendicular to the wall.

Example 9.1. *A uniform ladder of length 3.25 m and weighing 250 N is placed against a smooth vertical wall with its lower end 1.25 m from the wall. The coefficient of friction between the ladder and floor is 0.3.*

What is the frictional force acting on the ladder at the point of contact between the ladder and the floor? Show that the ladder will remain in equilibrium in this position.

Solution. Given: Length of the ladder (l) = 3.25 m; Weight of the ladder (w) = 250 N; Distance between the lower end of ladder and wall = 1.25 m and coefficient of friction between the ladder and floor (μ_f) = 0.3.

Frictional force acting on the ladder.

The forces acting on the ladder are shown in Fig. 9.2.

let F_f = Frictional force acting on the ladder at the Point of contact between the ladder and floor, and

R_f = Normal reaction at the floor.

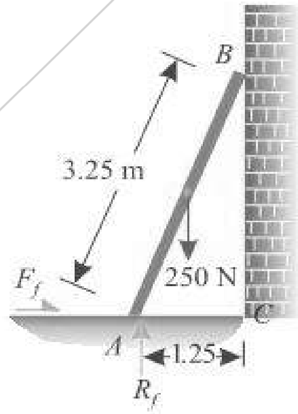


Fig. 9.2.

Since the ladder is placed against a smooth vertical wall, therefore there will be no friction at the point of contact between the ladder and wall. Resolving the forces vertically,

$$R_f = 250 \text{ N}$$

From the geometry of the figure, we find that

$$BC = \sqrt{(3.25)^2 - (1.25)^2} = 3.0 \text{ m}$$

Taking moments about B and equating the same,

$$F_f \times 3 = (R_f \times 1.25) - (250 \times 0.625) = (250 \times 1.25) - 156.3 = 156.2 \text{ N}$$

$$\therefore \quad F_f = \frac{156.2}{3} = 52.1 \text{ N} \quad \textbf{Ans.}$$

Example 9.3. *A uniform ladder of 4 m length rests against a vertical wall with which it makes an angle of 45°. The coefficient of friction between the ladder and the wall is 0.4 and that between ladder and the floor is 0.5. If a man, whose weight is one-half of that of the ladder ascends it, how high will it be when the ladder slips?*

Solution. Given: Length of the ladder (l) = 4 μ; Angle which the ladder makes with the horizontal (α) = 45°; Coefficient of friction between the ladder and the wall (μ_w) = 0.4 and coefficient of friction between the ladder and the floor (μ_f) = 0.5.

The forces acting on the ladder are shown in Fig. 9.4.

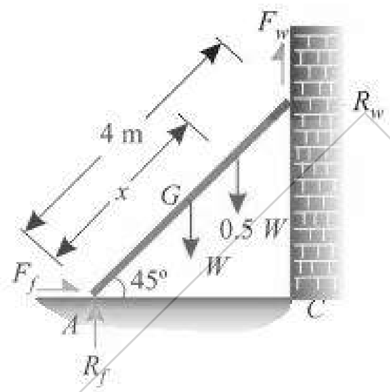


Fig. 9.4.

Let x = Distance between A and the man, when the ladder is at the point of slipping.

W = Weight of the ladder, and

R_f = Normal reaction at floor.

∴ Weight of the man

$$= \frac{W}{2} = 0.5\,W$$

We know that frictional force at the floor,

$$F_f = \mu_f\,R_f = 0.5\,R_f \quad ...(i)$$

and frictional force at the wall,

$$F_w = \mu_w\,R_w = 0.4\,R_w \quad ...(ii)$$

Resolving the forces vertically,

$$R_f + F_w = W + 0.5W = 1.5\,W \quad ...(iii)$$

and now resolving the forces horizontally,

$$R_w = F_f = 0.5\,R_f \quad \text{or} \quad R_f = 2R_w$$

Now substituting the values of R_f and F_w in equation (*iii*),

$$2R_w + 0.4\,R_w = 1.5\,W$$

$$\therefore \quad R_w = \frac{1.5\,W}{2.4} = 0.625\,W$$

and $$F_w = 0.4\,R_w = 0.4 \times 0.625\,W = 0.25\,W \quad ...(iv)$$

Taking moments about A and equating the same,

$$(W \times 2\cos 45°) + (0.5\,W \times x\cos 45°)$$
$$= (R_w \times 4\sin 45°) + (F_w \times 4\cos 45°)$$

Substituting values of R_w *and* F_w from equations (*iii*) and (*iv*) in the above equation,

$$(W \times 2\cos 45°) + (0.5\,W \times x\cos 45°)$$
$$= (0.625\,W \times 4\sin 45°) + (0.25\,W \times 4\cos 45°)$$

Dividing both sides by ($W\sin 45°$)*,

$$2 + 0.5\,x = 2.5 + 1 = 3.5$$

$$\therefore \quad x = \frac{3.5 - 2}{0.5} = 3.0\text{ m} \qquad \textbf{Ans.}$$

Wedge Friction

A wedge is, usually, of a triangular or trapezoidal in cross-section. It is, generally, used for slight adjustments in the position of a body i.e. for tightening fits or keys for shafts. Sometimes, a wedge is also used for lifting heavy weights as shown in Fig. 9.10.

It will be interesting to know that the problems on wedges are basically the problems of equilibrium on inclined planes. Thus these problems may be solved either by the equilibrium method or by applying Lami's theorem. Now consider a wedge ABC, which is used to lift the body DEFG.

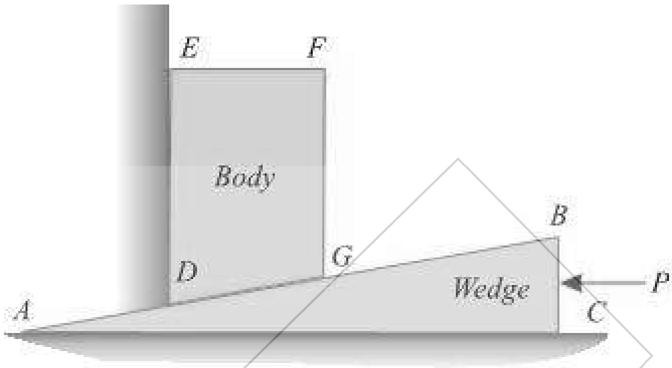


Fig. 9.10.

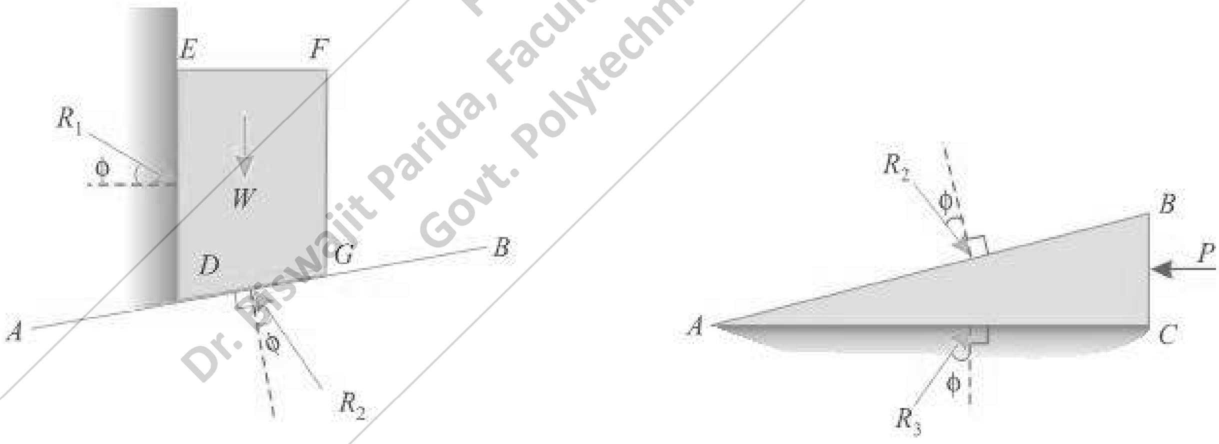
Let W = Weight of the body DEFG,

P = Force required to lift the body, and

μ = Coefficient of friction on the planes AB, AC and DE such that

tan φ = μ.

A little consideration will show that when the force is sufficient to lift the body, the sliding will take place along three planes AB, AC and DE will also occur as shown in Fig. 9.11 (a) and (b).



(a) Forces on the body DEFG

(a) Forces on the wedge ABC

Fig. 9.11.

The three reactions and the horizontal force (P) may now be found out either by graphical method or analytical method as discussed below:

Graphical method

a. First of all, draw the space diagram for the body DEFG and the wedge ABC as shown in Fig. 9.12 (a). Now draw the reactions R_1, R_2 and R_3 at angle f with normal to the faces DE, AB and AC respectively (such that $tan\ \varphi = \mu$).

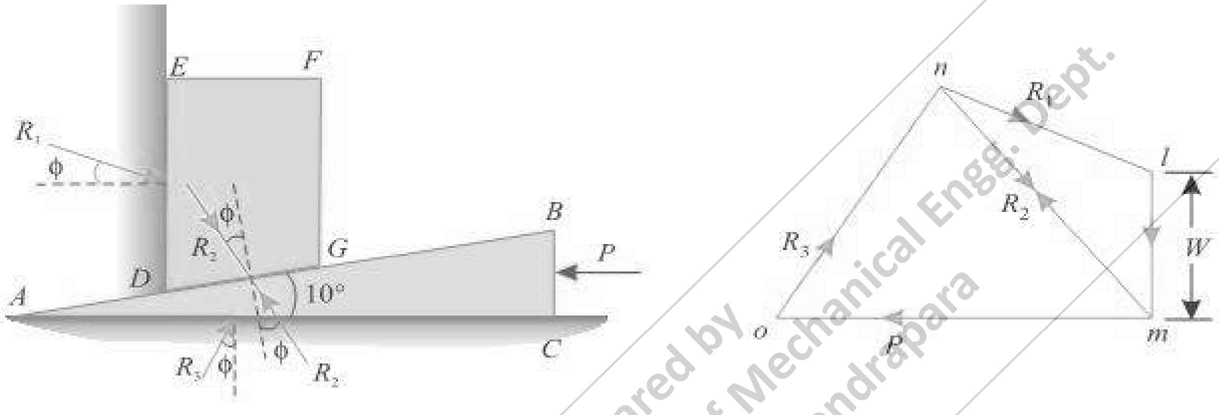
b. Now consider the equilibrium of the body DEFG. We know that the body is in equilibrium under the action of

(a) Its own weight (W) acting downwards

(b) Reaction R_1 on the face DE, and

(c) Reaction R_2 on the face AB.

Now, in order to draw the vector diagram for the above mentioned three forces, take some suitable point *l* and draw a vertical line lm parallel to the line of action of the weight (*W*) and cut off lm equal to the weight of the body to some suitable scale. Through *l* draw a line parallel to the reaction R_1. Similarly, through m draw a line parallel to the reaction R_2, meeting the first line at n as shown in Fig. 9.12 (b).



(*a*) Space diagram (*b*) Vector diagram

Fig. 9.12.

c. Now consider the equilibrium of the wedge ABC. We know that it is equilibrium under the action of

i) Force acting on the wedge (P),

ii) Reaction R_2 on the face AB, and

iii) Reaction R_3 on the face AC.

Now, in order to draw the vector diagram for the above mentioned three forces, through *m* draw a horizontal line parallel to the force (*P*) acting on the wedge. Similarly, through *n* draw a line parallel to the reaction R_3 meeting the first line at O as shown in Fig. 9.12 (b).

d. Now the force (P) required on the wedge to raise the load will be given by *mo* to the scale.

Analytical method

a. First of all, consider the equilibrium of the body DEFG. And resolve the forces W, R_1 and R_2 horizontally as well as vertically.

b. Now consider the equilibrium of the wedge ABC. And resolve the forces P, R_2 and R_3 horizontally as well as vertically.

Example 9.6. *A block weighing 1500 N, overlying a 10° wedge on a horizontal floor and leaning against a vertical wall, is to be raised by applying a horizontal force to the wedge.*

Assuming the coefficient of friction between all the surface in contact to be 0.3, determine the minimum horizontal force required to raise the block.

Solution. Given: Weight of the block (W) = 1500 N; Angle of the wedge (α) = 10° and coefficient of friction between all the four surfaces of contact (μ) = 0.3 = $\tan \phi$ or ϕ = 16.7°.

Let P = Minimum horizontal force required to raise the block.

The example may be solved graphically or analytically. But we shall solve it by both the methods.

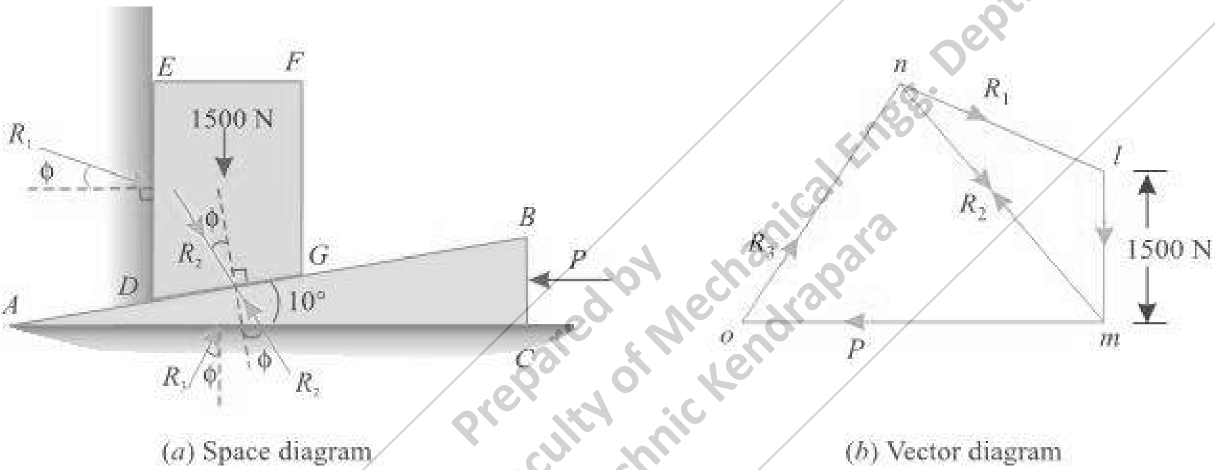


Fig. 9.13.

Graphical method

1. First of all, draw the space diagram for the block $DEFG$ and the wedge ABC as shown in Fig. 9.13 (a). Now draw reactions R_1, R_2 and R_3 at angles of ϕ (*i.e.* 16.7° with normal to the faces DE, AB and AC respectively.
2. Take some suitable point l, and draw vertical line lm equal to 1500 N to some suitable scale (representing the weight of the block). Through l, draw a line parallel to the reaction R_1. Similarly, through m draw another line parallel to the reaction R_2 meeting the first line at n.
3. Now through m, draw a horizontal line (representing the horizontal force P). Similarly, through n draw a line parallel to the reaction R_3 meeting the first line at O as shown in Fig. 9.13(b).
4. Now measuring mo to the scale, we find that the required horizontal force

$$P = 1420 \text{ N.} \quad \textbf{Ans.}$$

Analytical method

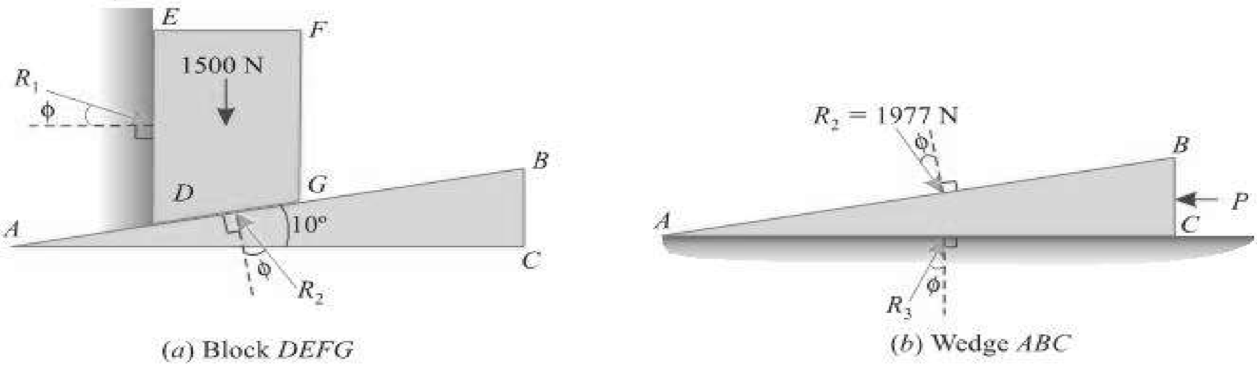


Fig. 9.14.

First of all, consider the equilibrium of the block. We know that it is in equilibrium under the action of the following forces as shown in Fig. 9.14 (*a*).

1. Its own weight 1500 N acting downwards.
2. Reaction R_1 on the face *DE*.
3. Reaction R_2 on the face *DG* of the block.

Resolving the forces horizontally,

$$R_1 \cos(16.7°) = R_2 \sin(10 + 16.7°) = R_2 \sin 26.7°$$

$$R_1 \times 0.9578 = R_2 \times 0.4493$$

or $$R_2 = 2.132\, R_1$$

and now resolving the forces vertically,

$$R_1 \times \sin(16.7°) + 1500 = R_2 \cos(10° + 16.7°) = R_2 \cos 26.7°$$

$$R_1 \times 0.2874 + 1500 = R_2 \times 0.8934 = (2.132\, R_1)0.8934$$

$$= 1.905\, R_1 \qquad ...(R_2 = 2.132\, R_1)$$

$$R_1(1.905 - 0.2874) = 1500$$

$$\therefore \qquad R_1 = \frac{1500}{1.6176} = 927.3 \text{ N}$$

and $$R_2 = 2.132\, R_1 = 2.132 \times 927.3 = 1977 \text{ N}$$

Now consider the equilibrium of the wedge. We know that it is in equilibrium under the action of the following forces as shown in Fig. 9.14 (*b*).

1. Reaction R_2 of the block on the wedge.
2. Force (*P*) acting horizontally, and
3. Reaction R_3 on the face *AC* of the wedge.

Resolving the forces vertically,

$$R_3 \cos 16.7° = R_2 \cos(10° + 16.7°) = R_2 \cos 26.7°$$

$$R_3 \times 0.9578 = R_2 \times 0.8934 = 1977 \times 0.8934 = 1766.2$$

$$\therefore \qquad R_3 = \frac{1766.2}{0.9578} = 1844 \text{ N}$$

and now resolving the forces horizontally,

$$P = R_2 \sin(10° + 16.7°) + R_3 \sin 16.7° = 1977 \sin 26.7° + 1844 \sin 16.7° \text{ N}$$

$$= (1977 \times 0.4493) + (1844 \times 0.2874) = 1418.3 \text{ N} \qquad \textbf{Ans.}$$

Advantages of Friction:

- Friction between our shoes and the ground helps in running and walking on the ground or the floor.
- We cannot fix nail in the wood or wall if there is no friction. It is friction which holds the nail.
- A horse cannot pull a cart unless friction furnishes him a secure Foothold.
- We can write with pencil or pen on the paper on the board using friction.
- Cars or other vehicles uses friction (Breaks) to stop the vehicle.
- Coffee mug stays on the dashboard
- Shuffling across a carpet to shock someone.

- Dragging of atmosphere with earth is possible.
- Helps to pevents the life on earth by burning astroids.

Disadvantages of Friction:

- The main disadvantage of friction is that it produces heat in various parts of machines.
- In this way some useful energy is wasted as heat energy and the machine efficiency is decreased.
- Due to friction we have to exert more power in machines.
- It counteracts movement and so it takes more energy to move.
- Due to friction, noise is also produced in machines.
- Due to friction, engines of automobiles consume more fuel which is a money loss.
- Friction makes wear and tear of the machinery
- Friction causes heat and cause energy loss.
- Forest fires are caused due to friction between branches of trees

Chapter-04: Centroid & Moment of Inertia

4.1 Centroid

- The centre of area of plane figures (like triangle, quadrilateral, circle etc.) is known as centroid.
- The method of finding out the centroid of a figure is the same as that of finding out the centre of gravity of a body.

Centre of Gravity by Geometrical Considerations

1. The centre of gravity of uniform rod is at its middle point.

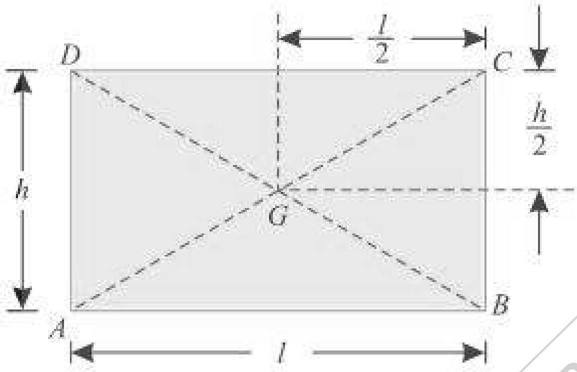


Fig. 6.1. Rectangle

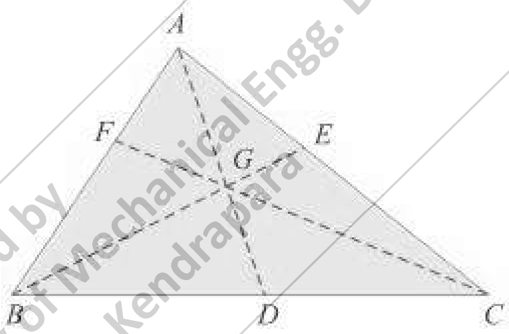


Fig. 6.2. Triangle

2. The centre of gravity of a rectangle (or a parallelogram) is at the point, where its diagonals meet each other. It is also a middle point of the length as well as the breadth of the rectangle as shown in Fig. 6.1.
3. The centre of gravity of a triangle is at the point, where the three medians (a median is a line connecting the vertex and middle point of the opposite side) of the triangle meet as shown in Fig. 6.2.
4. The centre of gravity of a trapezium with parallel sides a and b is at a distance of $\frac{h}{3} \times \left(\frac{b + 2a}{b + a}\right)$ measured form the side b as shown in Fig. 6.3.
5. The centre of gravity of a semicircle is at a distance of $\frac{4r}{3\pi}$ from its base measured along the vertical radius as shown in Fig. 6.4.

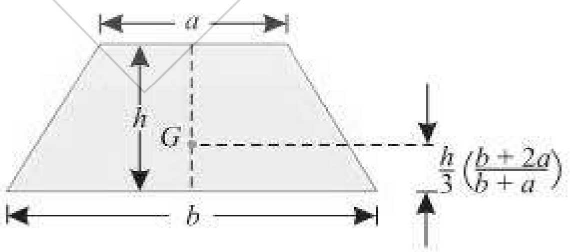


Fig. 6.3. Trapezium

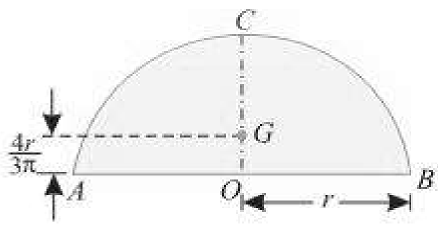


Fig. 6.4. Semicircle

6. The centre of gravity of a circular sector making semi-vertical angle α is at a distance of $\frac{2r}{3}\frac{\sin\alpha}{\alpha}$ from the centre of the sector measured along the central axis as shown in Fig. 6.5.

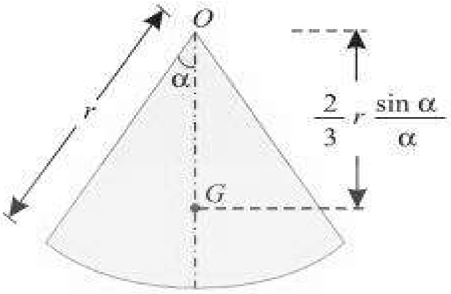


Fig. 6.5. Circular sector

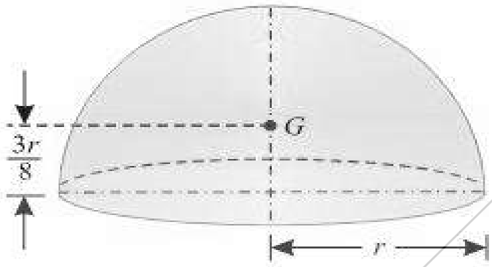


Fig. 6.6. Hemisphere

7. The centre of gravity of a cube is at a distance of $\frac{l}{2}$ from every face (where l is the length of each side).
8. The centre of gravity of a sphere is at a distance of $\frac{d}{2}$ from every point (where d is the diameter of the sphere).
9. The centre of gravity of a hemisphere is at a distance of $\frac{3r}{8}$ from its base, measured along the vertical radius as shown in Fig. 6.6.

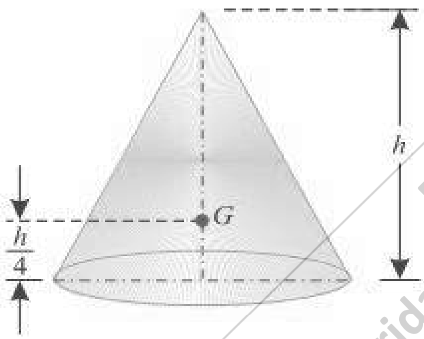


Fig. 6.7. Right circular solid cone

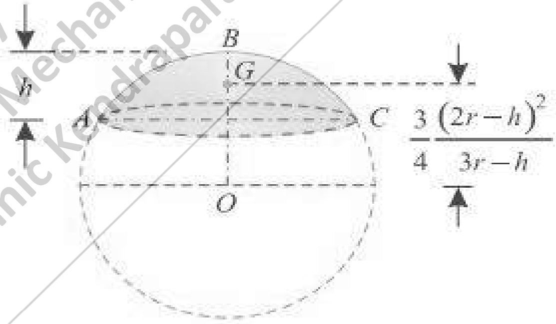


Fig.6.8. Segment of a sphere

10. The centre of gravity of right circular solid cone is at a distance of $\frac{h}{4}$ from its base, measured along the vertical axis as shown in Fig. 6.7.
11. The centre of gravity of a segment of sphere of a height h is at a distance of $\frac{3}{4} \frac{(2r-h)^2}{(3r-h)}$ from the centre of the sphere measured along the height. as shown in Fig. 6.8.

Centre of Gravity by Moments

The centre of gravity of a body may also be found out by moments as discussed below:

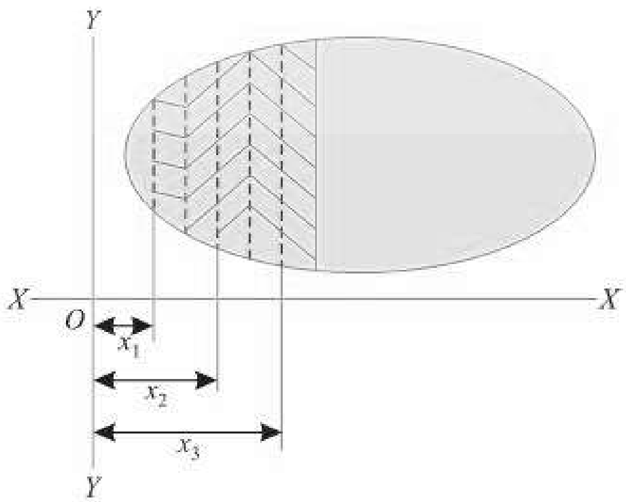


Fig. 6.9. Centre of gravity by moments

Consider a body of mass M whose centre of gravity is required to be found out. Divide the body into small masses, whose centres of gravity are known as shown in Fig. 6.9. Let m_1, m_2, m_3....; etc. be the masses of the particles and (x_1, y_1), (x_2, y_2), (x_3, y_3), ... be the co-ordinates of the centres of gravity from a fixed point O as shown in Fig. 6.9.

Let $\bar{x}$ and $\bar{y}$ be the co-ordinates of the centre of gravity of the body. From the principle of moments, we know that

$$M\bar{x} = m_1 x_1 + m_2 x_2 + m_3 x_3 \,.....$$

or
$$\bar{x} = \frac{\Sigma mx}{M}$$

Similarly
$$\bar{y} = \frac{\Sigma my}{M},$$

where
$$M = m_1 + m_2 + m_3 + \,.....$$

Axis of Reference

- The centre of gravity of a body is always calculated with reference to some assumed axis known as axis of reference.
- The axis of reference, of plane figures, is generally taken as the lowest line of the figure for calculating $\bar{y}$ and the left line of the figure for calculating $\bar{x}$.

Centre of Gravity of Plane Figures

The plane geometrical figures (such as T-section, I-section, L-section etc.) have only areas but no mass. The centre of area of such figures is known as centroid, and coincides with the centre of gravity of the figure.

Let $\bar{x}$ and $\bar{y}$ be the co-ordinates of the centre of gravity with respect to some axis of reference, then

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \,........}{a_1 + a_2 + a_3}$$

and
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \,........}{a_1 + a_2 + a_3 + \,...}$$

where a_1, a_2, a_3........ etc., are the areas into which the whole figure is divided x_1, x_2, x_3 ..... etc., are the respective co-ordinates of the areas a_1, a_2, a_3....... on X-X axis with respect to same axis of reference.

y_1, y_2, y_3....... etc., are the respective co-ordinates of the areas a_1, a_2, a_3....... on Y-Y axis with respect to same axis of the reference.

Note. While using the above formula, x_1, x_2, x_3 ..... or y_1, y_2, y_3 or $\bar{x}$ and $\bar{y}$ must be measured from the same axis of reference (or point of reference) and on the same side of it. However, if the figure is on both sides of the axis of reference, then the distances in one direction are taken as positive and those in the opposite directions must be taken as negative.

Centre of Gravity of Composite Figures

The plane geometrical figures (such as T-section, I-section, L-section etc.) have only areas but no mass.

a) ***Centre of Gravity of Symmetrical Sections:*** The given section is symmetrical about X-X axis or Y-Y axis. In such cases, we have only to calculate either $\bar{x}$ or $\bar{y}$. This is due to the reason that the centre of gravity of the body will lie on the axis of symmetry.

Example 6.1. *Find the centre of gravity of a 100 mm × 150 mm × 30 mm T-section.*

Solution. As the section is symmetrical about *Y-Y* axis, bisecting the web, therefore its centre of gravity will lie on this axis. Split up the section into two rectangles *ABCH* and *DEFG* as shown in Fig 6.10.

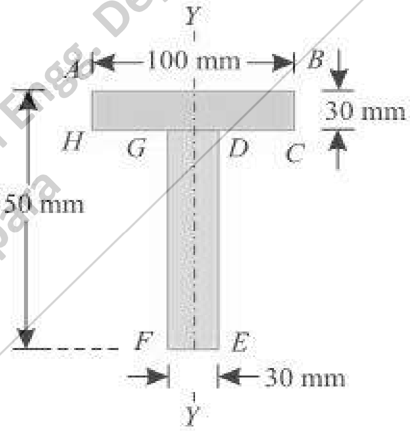


Fig. 6.10.

Let bottom of the web *FE* be the axis of reference.

(*i*) *Rectangle ABCH*

$$a_1 = 100 \times 30 = 3000 \text{ mm}^2$$

and $$y_1 = \left(150 - \frac{30}{2}\right) = 135 \text{ mm}$$

(*ii*) *Rectangle DEFG*

$$a_2 = 120 \times 30 = 3600 \text{ mm}^2$$

and $$y_2 = \frac{120}{2} = 60 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the flange *FE*,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} \text{ mm}$$

$$= 94.1 \text{ mm} \quad \textbf{Ans.}$$

Example 6.3. *An I-section has the following dimensions in mm units :*

Bottom flange = *300 × 100*

Top flange = *150 × 50*

Web = *300 × 50*

Determine mathematically the position of centre of gravity of the section.

Solution. As the section is symmetrical about *Y-Y* axis, bisecting the web, therefore its centre of gravity will lie on this axis. Now split up the section into three rectangles as shown in Fig. 6.12.

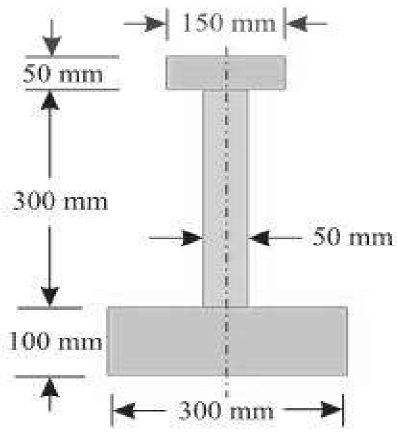


Fig. 6.12.

Let bottom of the bottom flange be the axis of reference.

(*i*) *Bottom flange*

$$a_1 = 300 \times 100 = 30\,000 \text{ mm}^2$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(*ii*) *Web*

$$a_2 = 300 \times 50 = 15\,000 \text{ mm}^2$$

and $$y_2 = 100 + \frac{300}{2} = 250 \text{ mm}$$

(*iii*) *Top flange*

$$a_3 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_3 = 100 + 300 + \frac{50}{2} = 425 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the flange,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(30\,000 \times 50) + (15\,000 \times 250) + (7500 \times 425)}{30\,000 + 15\,000 + 7500} = 160.7 \text{ mm} \quad \textbf{Ans.}$$

Example 6.2. *Find the centre of gravity of a channel section 100 mm × 50 mm × 15 mm.*

Solution. As the section is symmetrical about *X-X* axis, therefore its centre of gravity will lie on this axis. Now split up the whole section into three rectangles *ABFJ*, *EGKJ* and *CDHK* as shown in Fig. 6.11.

Let the face *AC* be the axis of reference.

(*i*) *Rectangle ABFJ*

$$a_1 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_1 = \frac{50}{2} = 25 \text{ mm}$$

(*ii*) *Rectangle EGKJ*

$$a_2 = (100 - 30) \times 15 = 1050 \text{ mm}^2$$

and $$x_2 = \frac{15}{2} = 7.5 \text{ mm}$$

(*iii*) *Rectangle CDHK*

$$a_3 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_3 = \frac{50}{2} = 25 \text{ mm}$$

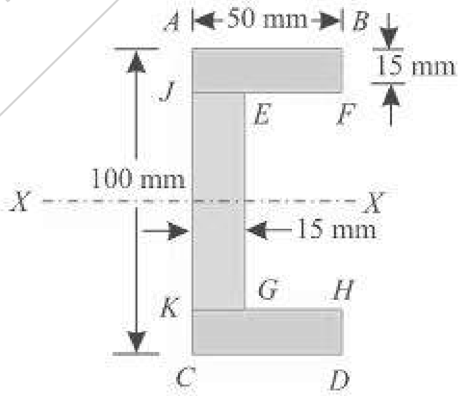


Fig. 6.11.

We know that distance between the centre of gravity of the section and left face of the section *AC*,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750} = 17.8 \text{ mm} \quad \textbf{Ans.}$$

b) Centre of Gravity of Un-Symmetrical Sections: The given section is not symmetrical either about X-X axis or Y-Y axis. In such cases, we have to calculate both $\bar{x}$ and $\bar{y}$. This is due to the reason that the centre of gravity of the body will lie on the axis of symmetry.

Example 6.4. *Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.*

Solution. As the section is not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the angle section. Split up the section into two rectangles as shown in Fig. 6.13.

Let left face of the vertical section and bottom face of the horizontal section be axes of reference.

(*i*) *Rectangle* 1

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

$$x_1 = \frac{20}{2} = 10 \text{ mm}$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

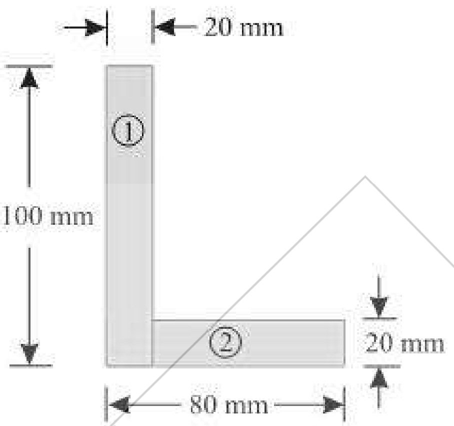


Fig. 6.13.

(*ii*) *Rectangle* 2

$$a_2 = (80 - 20) \times 20 = 1200 \text{ mm}^2$$

$$x_2 = 20 + \frac{60}{2} = 50 \text{ mm}.$$

and $$y_2 = \frac{20}{2} = 10 \text{ mm}$$

We know that distance between centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \text{ mm} \quad \textbf{Ans.}$$

Similarly, distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35 \text{ mm} \quad \textbf{Ans.}$$

Example 6.5. *A uniform lamina shown in Fig. 6.14 consists of a rectangle, a circle and a triangle.*

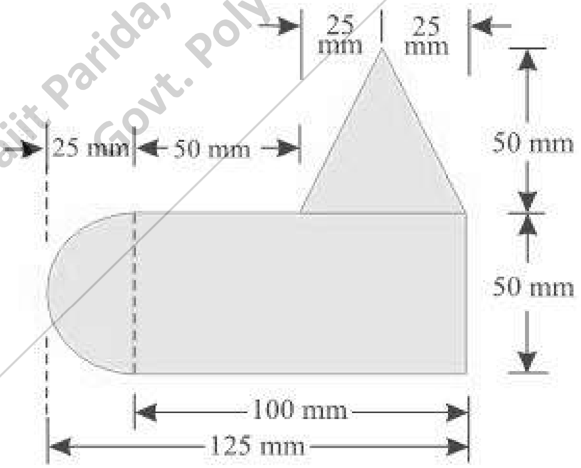


Fig. 6.14.

Determine the centre of gravity of the lamina. All dimensions are in mm.

Solution. As the section is not symmetrical about any axis, therefore we have to find out the values of both $\bar{x}$ and $\bar{y}$ for the lamina.

Let left edge of circular portion and bottom face rectangular portion be the axes of reference.

(*i*) *Rectangular portion*

$$a_1 = 100 \times 50 = 5000 \text{ mm}^2$$

$$x_1 = 25 + \frac{100}{2} = 75 \text{ mm}$$

and $$y_1 = \frac{50}{2} = 25 \text{ mm}$$

(ii) *Semicircular portion*

$$a_2 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2}(25)^2 = 982 \text{ mm}^2$$

$$x_2 = 25 - \frac{4r}{3\pi} = 25 - \frac{4 \times 25}{3\pi} = 14.4 \text{ mm}$$

and $$y_2 = \frac{50}{2} = 25 \text{ mm}$$

(iii) *Triangular portion*

$$a_3 = \frac{50 \times 50}{2} = 1250 \text{ mm}^2$$

$$x_3 = 25 + 50 + 25 = 100 \text{ mm}$$

and $$y_3 = 50 + \frac{50}{3} = 66.7 \text{ mm}$$

We know that distance between centre of gravity of the section and left edge of the circular portion,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 75) + (982 \times 14.4) + (1250 \times 100)}{5000 + 982 + 1250}$$

$$= 71.1 \text{ mm} \quad \textbf{Ans.}$$

c) Centre of Gravity of Sections with Cut Out Holes: The centre of gravity of such a section is found out by considering the main section, first as a complete one, and then deducting the area of the cut out hole i.e., by taking the area of the cut out hole as negative. Now substituting a_2 (i.e., the area of the cut out hole) as negative, in the general equation for the centre of gravity, we get.

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} \quad \text{and} \quad \bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

Example 6.12. *A square hole is punched out of circular lamina, the digonal of the square being the radius of the circle as shown in Fig.6.22. Find the centre of gravity of the remainder, if r is the radius of the circle.*

Solution. As the section is symmetrical about *X-X* axis, therefore its centre of gravity will lie on this axis. Let *A* be the point of reference.

(i) *Main circle*

$$a_1 = \pi r^2$$

and $$x_1 = r$$

(ii) *Cut out square*

$$a_2 = \frac{r \times r}{2} = 0.5\, r^2$$

and $$x_2 = r + \frac{r}{2} = 1.5 r$$

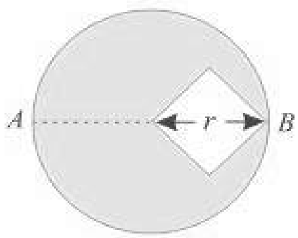


Fig. 6.22.

We know that distance between centre of gravity of the section and *A*,

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = \frac{(\pi r^2 \times r) - (0.5 r^2 \times 1.5 r)}{\pi r^2 - 0.5 r^2}$$

$$= \frac{r^3(\pi - 0.75)}{r^2(\pi - 0.5)} = \frac{r(\pi - 0.75)}{\pi - 0.5} \quad \textbf{Ans.}$$

Example 6.14. *A semicircular area is removed from a trapezium as shown in Fig.6.24 (dimensions in mm)*

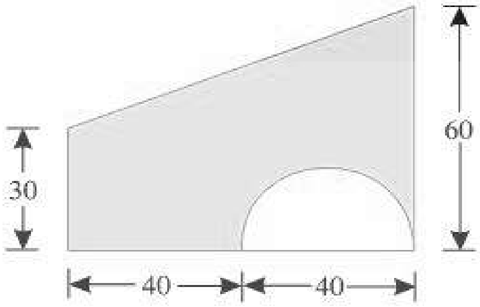


Fig. 6.24.

Determine the centroid of the remaining area (shown hatched).

Solution. As the section in not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the area. Split up the area into three parts as shown in Fig. 6.25. Let left face and base of the trapezium be the axes of reference.

(*i*) *Rectangle*

$$a_1 = 80 \times 30 = 2400 \text{ mm}^2$$

$$x_1 = \frac{80}{2} = 40 \text{ mm}$$

and $$y_1 = \frac{30}{2} = 15 \text{ mm}$$

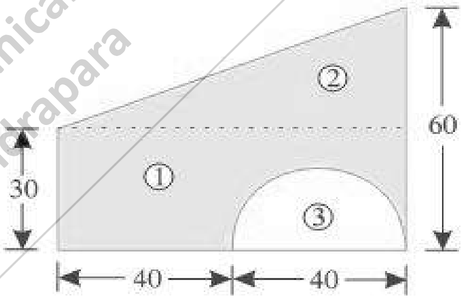


Fig. 6.25.

(*ii*) *Triangle*

$$a_2 = \frac{80 \times 30}{2} = 1200 \text{ mm}^2$$

$$x_2 = \frac{80 \times 2}{3} = 53.3 \text{ mm}$$

and $$y_2 = 30 + \frac{30}{3} = 40 \text{ mm}$$

(*iii*) *Semicircle*

$$a_3 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2}(20)^2 = 628.3 \text{ mm}^2$$

$$x_3 = 40 + \frac{40}{2} = 60 \text{ mm}$$

and $$y_3 = \frac{4r}{3\pi} = \frac{4 \times 20}{3\pi} = 8.5 \text{ mm}$$

We know that distance between centre of gravity of the area and left face of trapezium,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 - a_3 x_3}{a_1 + a_2 - a_3} = \frac{(2400 \times 40) + (1200 \times 53.3) - (628.3 \times 60)}{2400 + 1200 - 628.3}$$

$= 41.1$ mm **Ans.**

Similarly, distance between centre of gravity of the area and base of the trapezium,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 - a_3 y_3}{a_1 + a_2 - a_3} = \frac{(2400 \times 15) + (1200 \times 40) - (628.3 \times 8.5)}{2400 + 1200 - 628.3}$$

$= 26.5$ mm **Ans.**

Example 6.13. *A semicircle of 90 mm radius is cut out from a trapezium as shown in Fig 6.23*

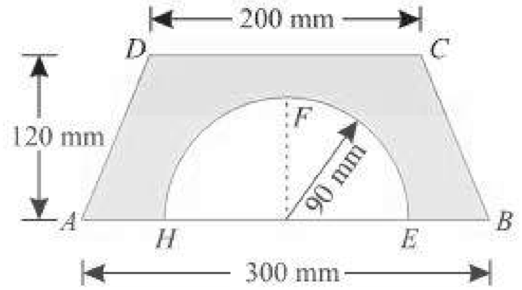


Fig. 6.23.

Find the position of the centre of gravity of the figure.

Solution. As the section is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis. Now consider two portions of the figure *viz.*, trapezium *ABCD* and semicircle *EFH*.

Let base of the trapezium *AB* be the axis of reference.

(*i*) *Trapezium ABCD*

$$a_1 = 120 \times \frac{200 + 300}{2} = 30\,000 \text{ mm}^2$$

and $$y_1 = \frac{120}{3} \times \left(\frac{300 + 2 \times 200}{300 + 200}\right) = 56 \text{ mm}$$

(*ii*) *Semicircle*

$$a_2 = \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \pi \times (90)^2 = 4050\,\pi \text{ mm}^2$$

and $$y_2 = \frac{4r}{3\pi} = \frac{4 \times 90}{3\pi} = \frac{120}{\pi} \text{ mm}$$

We know that distance between centre of gravity of the section and *AB*,

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{(30\,000 \times 56) - \left(4050\pi \times \frac{120}{\pi}\right)}{30\,000 - 4050\pi} \text{ mm}$$

$$= 69.1 \text{ mm} \quad \textbf{Ans.}$$

4.2 Moment of Inertia

- moment of a force (P) about a point, is the product of the force and perpendicular distance (x) between the point and the line of action of the force (i.e. P.x). This moment is also called first moment of force..
- If this moment is again multiplied by the perpendicular distance (x) between the point and the line of action of the force i.e. P.x (x) = Px^2, then this quantity is called moment of the moment of a force or second moment of force or moment of inertia (briefly written as M.I.).
- Sometimes, instead of force, area or mass of a figure or body is taken into consideration.

Moment Of Inertia of a Plane Area

Consider a plane area, whose moment of inertia is required to be found out. Split up the whole area into a number of small elements.

Let $a_1, a_2, a_3, ...$ = Areas of small elements, and

$r_1, r_2, r_3, ...$ = Corresponding distances of the elements from the line about which the moment of inertia is required to be found out.

Now the moment of inertia of the area,

$$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + ...$$
$$= \sum a\, r^2$$

Units

The units of moment of inertia of a plane area depend upon the units of the area and the length. e.g.,

1. If area is in m^2 and the length is also in m, the moment of inertia is expressed in m^4.
2. If area in mm^2 and the length is also in mm, then moment of inertia is expressed in mm^4

Methods for Moment Of Inertia

a) ***Moment Of Inertia by ROUTH'S Rule:*** It The Routh's Rule states, if a body is symmetrical about three mutually perpendicular axes, then the moment of inertia, about any one axis passing through its centre of gravity is given by:

$$I = \frac{A\ (\text{or } M) \times S}{3} \quad \text{... (For a Square or Rectangular Lamina)}$$

$$I = \frac{A\ (\text{or } M) \times S}{4} \quad \text{... (For a Circular or Elliptical Lamina)}$$

$$I = \frac{A\ (\text{or } M) \times S}{5} \quad \text{... (For a Spherical Body)}$$

where A = Area of the plane area

M = Mass of the body, and

S = Sum of the squares of the two semi-axis, other than the axis, about which the moment of inertia is required to be found out.

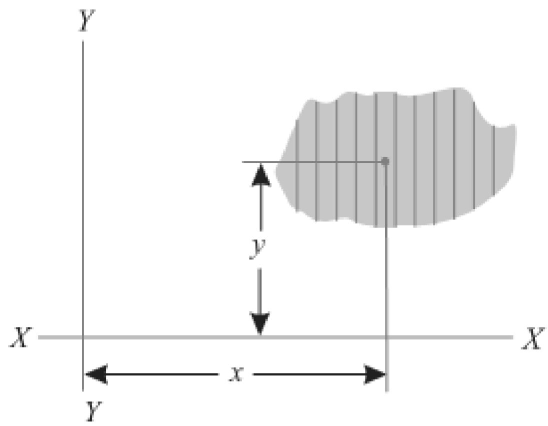
b) Moment Of Inertia by Integration:

Consider a plane figure, whose moment of inertia is required to be found out about X-X axis and Y-Y axis as shown in Fig 7.1. Let us divide the whole area into a no. of strips. Consider one of these strips.

Let dA = Area of the strip

x = Distance of the centre of gravity of the strip on X-X axis and

y = Distance of the centre of gravity of the strip on Y-Y axis.



We know that the moment of inertia of the strip about Y-Y axis

$= dA \cdot x^2$

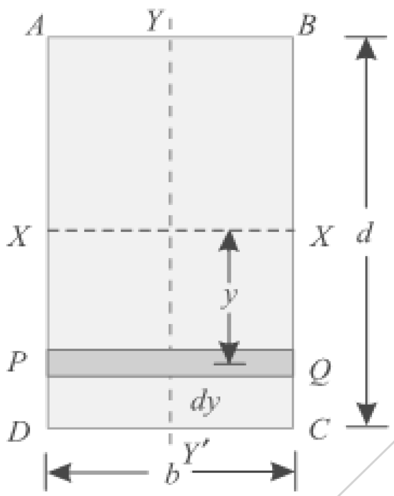
Now the moment of inertia of the whole area may be found out by integrating above equation. i.e.,

$I_{YY} = \sum dA \cdot x^2$

Similarly $I_{XX} = \sum dA \cdot y^2$

Moment Of Inertia of a Rectangular Section

Consider a rectangular section ABCD as shown in Fig. 7.2 whose moment of inertia is required to be found out.



Let b = Width of the section and

d = Depth of the section.

Now consider a strip PQ of thickness dy parallel to X-X axis and at a distance y from it as shown in the figure

∴ Area of the strip = b.dy

We know that moment of inertia of the strip about X-X axis,

$= \text{Area} \times y^2 = (b.\ dy)\ y^2$

$= b.\ y^2.\ dy$

Now *moment of inertia of the whole section may be found out by integrating the above equation for the whole length of the lamina i.e. from –d/2 to +d/2.

$$I_{xx} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} b.y^2.dy = b \int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2 \cdot dy$$

$$= b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{+\frac{d}{2}} = b\left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right] = \frac{bd^3}{12}$$

Similarly, $$I_{YY} = \frac{db^3}{12}$$

Example 7.1. *Find the moment of inertia of a rectangular section 30 mm wide and 40 mm deep about X-X axis and Y-Y axis.*

Solution. Given: Width of the section (b) = 30 mm and depth of the section (d) = 40 mm.

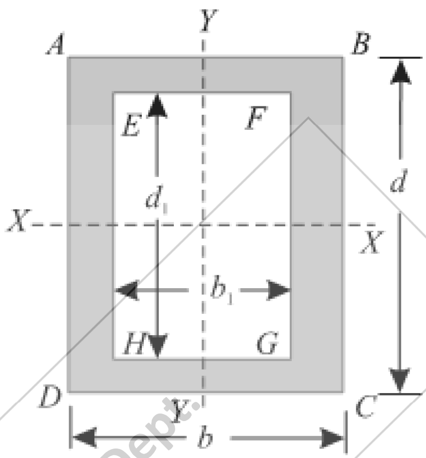
We know that moment of inertia of the section about an axis passing through its centre of gravity and parallel to X-X axis,

$$I_{XX} = \frac{bd^3}{12} = \frac{30 \times (40)^3}{12} = 160 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Similarly $$I_{YY} = \frac{db^3}{12} = \frac{40 \times (30)^3}{12} = 90 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Moment Of Inertia of a Hollow Rectangular Section

Consider a hollow rectangular section, in which ABCD is the main section and EFGH is the cut out section as shown in Fig 7.3



Let

b = Breadth of the outer rectangle,

d = Depth of the outer rectangle and

b_1, d_1 = Corresponding values for the cut out rectangle.

We know that the moment of inertia, of the outer rectangle ABCD about X-X axis

$$= \frac{bd^3}{12} \quad ...(i)$$

and moment of inertia of the cut out rectangle *EFGH* about *X-X* axis

$$= \frac{b_1 d_1^3}{12} \quad ...(ii)$$

$\therefore$ M.I. of the hollow rectangular section about *X-X* axis,

$$I_{XX} = \text{M.I. of rectangle } ABCD - \text{M.I. of rectangle } EFGH$$

$$= \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$$

Similarly, $$I_{YY} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12}$$

Example 7.2. *Find the moment of inertia of a hollow rectangular section about its centre of gravity if the external dimensions are breadth 60 mm, depth 80 mm and internal dimensions are breadth 30 mm and depth 40 mm respectively.*

Solution. Given: External breadth (b) = 60 mm; External depth (d) = 80 mm ; Internal breadth (b_1) = 30 mm and internal depth (d_1) = 40 mm.

We know that moment of inertia of hollow rectangular section about an axis passing through its centre of gravity and parallel to *X-X* axis,

$$I_{XX} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12} = \frac{60\,(80)^3}{12} - \frac{30\,(40)^3}{12} = 2400 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Similarly, $$I_{YY} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12} = \frac{80\,(60)^3}{12} - \frac{40\,(30)^3}{12} = 1350 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Perpendicular Axis Theorem

It states, If I_{XX} and I_{YY} be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia I_{ZZ} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:

$$I_{ZZ} = I_{XX} + I_{YY}$$

Proof :

Consider a small lamina (P) of area da having co-ordinates as x and y along OX and OY two mutually perpendicular axes on a plane section as shown in Fig. 7.4.

Now consider a plane OZ perpendicular to OX and OY. Let (r) be the distance of the lamina (P) from Z-Z axis such that $OP = r$.

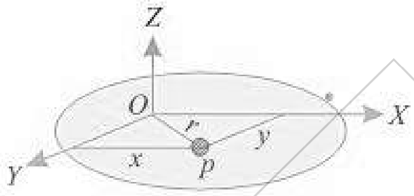


Fig. 7.4. Theorem of perpendicular axis.

From the geometry of the figure, we find that

$$r^2 = x^2 + y^2$$

We know that the moment of inertia of the lamina P about X-X axis,

$$I_{XX} = da.\, y^2 \qquad ...[\because I = \text{Area} \times (\text{Distance})^2]$$

Similarly, $I_{YY} = da.\, x^2$

and $I_{ZZ} = da.\, r^2 = da\,(x^2 + y^2) \qquad ...(\because r^2 = x^2 + y^2)$

$$= da.\, x^2 + da.\, y^2 = I_{YY} + I_{XX}$$

Moment Of Inertia of a Circular Section

Consider a circle $ABCD$ of radius (r) with centre O and X-X' and Y-Y' be two axes of reference through O as shown in Fig. 7.5.

Now consider an elementary ring of radius x and thickness dx. Therefore area of the ring,

$$da = 2\,\pi\, x.\, dx$$

and moment of inertia of ring, about X-X axis or Y-Y axis

$$= \text{Area} \times (\text{Distance})^2$$
$$= 2\,\pi\, x.\, dx \times x^2$$
$$= 2\,\pi\, x^3.\, dx$$

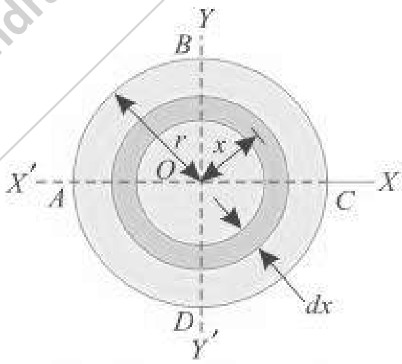


Fig. 7.5. Circular section.

Now moment of inertia of the whole section, about the central axis, can be found out by integrating the above equation for the whole radius of the circle *i.e.*, from 0 to r.

$$\therefore \quad I_{ZZ} = \int_0^r 2\,\pi\, x^3\,.\, dx = 2\pi \int_0^r x^3\,.\, dx$$

$$I_{ZZ} = 2\pi \left[\frac{x^4}{4}\right]_0^r = \frac{\pi}{2}(r)^4 = \frac{\pi}{32}(d)^4 \qquad ...\left(\text{substituting } r = \frac{d}{2}\right)$$

We know from the Theorem of Perpendicular Axis that

$$I_{XX} + I_{YY} = I_{ZZ}$$

$$\therefore \quad *I_{XX} = I_{YY} = \frac{I_{ZZ}}{2} = \frac{1}{2} \times \frac{\pi}{32}(d)^4 = \frac{\pi}{64}(d)^4$$

Example 7.3. *Find the moment of inertia of a circular section of 50 mm diameter about an axis passing through its centre.*

Solution. Given: Diameter (d) = 50 mm

We know that moment of inertia of the circular section about an axis passing through its centre,

$$I_{XX} = \frac{\pi}{64}(d)^4 = \frac{\pi}{64} \times (50)^4 = 307 \times 10^3 \text{ mm}^4 \qquad \textbf{Ans.}$$

Moment Of Inertia of a Hollow Circular Section

Consider a hollow circular section as shown in Fig.7.6, whose moment of inertia is required to be found out.

Let D = Diameter of the main circle, and

d = Diameter of the cut out circle.

We know that the moment of inertia of the main circle about X-X axis

$$= \frac{\pi}{64}(D)^4$$

and moment of inertia of the cut-out circle about X-X axis

$$= \frac{\pi}{64}(d)^4$$

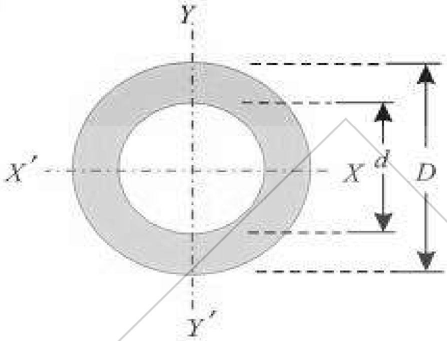


Fig. 7.6. Hollow circular section.

$\therefore$ Moment of inertia of the hollow circular section about X-X axis,

I_{XX} = Moment of inertia of main circle – Moment of inertia of cut out circle,

$$= \frac{\pi}{64}(D)^4 - \frac{\pi}{64}(d)^4 = \frac{\pi}{64}(D^4 - d^4)$$

Similarly, $I_{YY} = \frac{\pi}{64}(D^4 - d^4)$

Example 7.4. *A hollow circular section has an external diameter of 80 mm and internal diameter of 60 mm. Find its moment of inertia about the horizontal axis passing through its centre.*

Solution. Given : External diameter (D) = 80 mm and internal diameter (d) = 60 mm.

We know that moment of inertia of the hollow circular section about the horizontal axis passing through its centre,

$$I_{XX} = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}[(80)^4 - (60)^4] = 1374 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Parallel Axis Theorem

It states, if the moment of inertia of a plane area about an axis through its centre of gravity is denoted by I_G, then moment of inertia of the area about any other axis AB, parallel to the first, and at a distance h from the centre of gravity is given by:

$$I_{AB} = I_G + ah^2$$

where I_{AB} = Moment of inertia of the area about an axis AB,

I_G = Moment of Inertia of the area about its centre of gravity

a = Area of the section, and

h = Distance between centre of gravity of the section and axis AB.

Proof

Consider a strip of a circle, whose moment of inertia is required to be found out about a line AB as shown in Fig. 7.7.

Let δa = Area of the strip

y = Distance of the strip from the centre of gravity the section and

h = Distance between centre of gravity of the section and the axis AB.

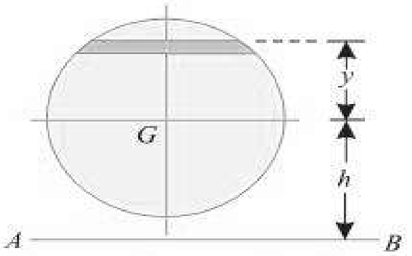


Fig. 7.7. Theorem of parallel axis.

We know that moment of inertia of the whole section about an axis passing through the centre of gravity of the section

$$= \delta a . y^2$$

and moment of inertia of the whole section about an axis passing through its centre of gravity,

$$I_G = \sum \delta a.\, y^2$$

∴ Moment of inertia of the section about the axis AB,

$$I_{AB} = \sum \delta a\,(h + y)^2 = \sum \delta a\,(h^2 + y^2 + 2\,h\,y)$$

$$= (\sum h^2.\, \delta a) + (\sum y^2.\, \delta a) + (\sum 2\,h\,y\,.\,\delta a)$$

$$= a\,h^2 + I_G + 0$$

It may be noted that $\sum h^2 \,.\, \delta a = a\,h^2$ and $\sum y^2 \,.\, \delta a = I_G$ [as per equation (i) above] and $\sum$ δa.y is the algebraic sum of moments of all the areas, about an axis through centre of gravity of the section and is equal to a. $\bar{y}$, where $\bar{y}$ is the distance between the section and the axis passing through the centre of gravity, which obviously is zero.

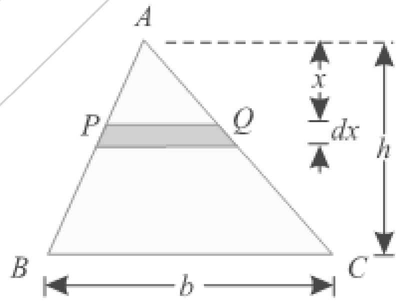
Moment Of Inertia of a Triangular Section

Consider a triangular section ABC whose moment of inertia is required to be found out.

Let b = Base of the triangular section and

h = Height of the triangular section.

Now consider a small strip PQ of thickness dx at a distance of x from the vertex A as shown in Fig. 7.8. From the geometry of the figure, we find that the two triangles APQ and ABC are similar. Therefore



$$\frac{PQ}{BC} = \frac{x}{h} \quad \text{or} \quad PQ = \frac{BC\,.\,x}{h} = \frac{bx}{h} \qquad (\because BC = \text{base} = b)$$

We know that area of the strip PQ

$$= \frac{bx}{h}\,.\,dx$$

and moment of inertia of the strip about the base BC

$$= \text{Area} \times (\text{Distance})^2 = \frac{bx}{h}\,dx\,(h - x)^2 = \frac{bx}{h}\,(h - x)^2\,dx$$

Now moment of inertia of the whole triangular section may be found out by integrating the above equation for the whole height of the triangle *i.e.*, from 0 to h.

$$I_{BC} = \int_0^h \frac{b\,x}{h}\,(h - x)^2\;dx$$

$$= \frac{b}{h}\int_0^h x\,(h^2 + x^2 - 2\,h\,x)\;dx$$

$$= \frac{b}{h}\int_0^h (xh^2 + x^3 - 2hx^2)\, dx$$

$$= \frac{b}{h}\left[\frac{x^2 h^2}{2} + \frac{x^4}{4} - \frac{2hx^3}{3}\right]_0^h = \frac{bh^3}{12}$$

We know that distance between centre of gravity of the triangular section and base BC,

$$d = \frac{h}{3}$$

∴ Moment of inertia of the triangular section about an axis through its centre of gravity and parallel to X-X axis,

$$I_G = I_{BC} - ad^2 \qquad ...(\because I_{XX} = I_G + a\,h^2)$$

$$= \frac{bh^3}{12} - \left(\frac{bh}{2}\right)\left(\frac{h}{3}\right)^2 = \frac{bh^3}{36}$$

Example. 7.5. *An isosceles triangular section ABC has base width 80 mm and height 60 mm. Determine the moment of inertia of the section about the centre of gravity of the section and the base BC.*

Solution. Given : Base width (b) = 80 mm and height (h) = 60 mm.

Moment of inertia about the centre of gravity of the section

We know that moment of inertia of triangular section about its centre of gravity,

$$I_G = \frac{bh^3}{36} = \frac{80 \times (60)^3}{36} = 480 \times 10^3 \text{ mm}^4$$

Moment of inertia about the base BC

We also know that moment of inertia of triangular section about the base BC,

$$I_{BC} = \frac{bh^3}{12} = \frac{80 \times (60)^3}{12} = 1440 \times 10^3 \text{ mm}^4$$

Exmple 7.6. *A hollow triangular section shown in Fig. 7.9 is symmetrical about its vertical axis.*

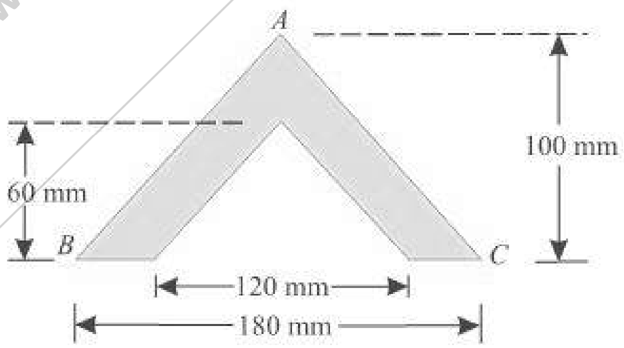


Fig. 7.9.

Find the moment of inertia of the section about the base BC.

Solution. Given : Base width of main triangle (B) = 180 mm; Base width of cut out triangle (b) = 120 mm; Height of main triangle (H) = 100 mm and height of cut out triangle (h) = 60 mm.

We know that moment of inertia of the triangular, section about the base BC,

$$I_{BC} = \frac{BH^3}{12} - \frac{bh^3}{12} = \frac{180 \times (100)^3}{12} - \frac{120 \times (60)^3}{12} \text{ mm}^4$$

$$= (15 \times 10^6) - (2.16 \times 10^6) = 12.84 \times 10^6 \text{ mm}^4 \qquad \textbf{Ans.}$$

Moment Of Inertia of a Semicircular Section

Consider a semicircular section ABC whose moment of inertia is required to be found out as shown in Fig. 7.10.

Let r = Radius of the semicircle.

We know that moment of inertia of the semicircular section about the base AC is equal to half the moment of inertia of the circular section about AC. Therefore moment of inertia of the semicircular section ABC about the base AC,

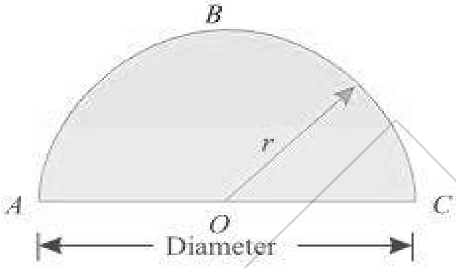


Fig. 7.10. Semicircular section ABC.

$$I_{AC} = \frac{1}{2} \times \frac{\pi}{64} \times (d)^4 = 0.393\ r^4$$

We also know that area of semicircular section,

$$a = \frac{1}{2} \times \pi r^2 \ \frac{\pi r^2}{2}$$

and distance between centre of gravity of the section and the base AC,

$$h = \frac{4r}{3\pi}$$

$\therefore$ Moment of inertia of the section through its centre of gravity and parallel to x-x axis,

$$I_G = I_{AC} - ah^2 = \left[\frac{\pi}{8} \times (r)^4\right] - \left[\frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right)^2\right]$$

$$= \left[\frac{\pi}{8} \times (r)^4\right] - \left[\frac{8}{9\pi} \times (r)^4\right] = 0.11\ r^4$$

Note. The moment of inertia about y-y axis will be the same as that about the base AC *i.e.*, 0.393 r^4.

Example 7.7. *Determine the moment of inertia of a semicircular section of 100 mm diameter about its centre of gravity and parallel to X-X and Y-Y axes.*

Solution. Given: Diameter of the section (d) = 100 mm or radius (r) = 50 mm

Moment of inertia of the section about its centre of gravity and parallel to X-X axis

We know that moment of inertia of the semicircular section about its centre of gravity and parallel to X-X axis,

$$I_{XX} = 0.11\ r^4 = 0.11 \times (50)^4 = 687.5 \times 10^3\ \text{mm}^4 \quad \textbf{Ans.}$$

Moment of inertia of the section about its centre of gravity and parallel to Y-Y axis.

We also know that moment of inertia of the semicircular section about its centre of gravity and parallel to Y-Y axis.

$$I_{YY} = 0.393\ r^4 = 0.393 \times (50)^4 = 2456 \times 10^3\ \text{mm}^4 \quad \textbf{Ans.}$$

Example 7.8. *A hollow semicircular section has its outer and inner diameter of 200 mm and 120 mm respectively as shown in Fig. 7.11.*

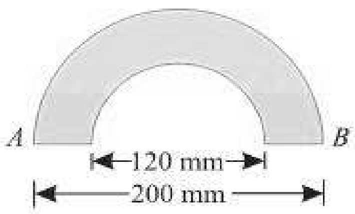


Fig. 7.11.

What is its moment of inertia about the base AB ?

Solution. Given: Outer diameter (D) = 200 mm or Outer Radius (R) = 100 mm and inner diameter (d) = 120 mm or inner radius (r) = 60 mm.

We know that moment of inertia of the hollow semicircular section about the base AB,

$$I_{AB} = 0.393\ (R^4 - r^4) = 0.393\ [(100)^4 - (60)^4] = 34.21 \times 10^6\ \text{mm}^4 \quad \textbf{Ans.}$$

Moment Of Inertia of a Composite Section

The moment of inertia of a composite section may be found out by the following steps:

- Split up the given section into plane areas (i.e., rectangular, triangular, circular etc., and find the centre of gravity of the section).
- Find the moments of inertia of these areas about their respective centres of gravity.
- Transfer these moment of inertia about the required axis (AB) by the Theorem of Parallel Axis, i.e.,

$$I_{AB} = I_G + ah^2$$

where I_G = Moment of inertia of a section about its centre of gravity and parallel to the axis.

a = Area of the section,

h = Distance between the required axis and centre of gravity of the section.

- The moments of inertia of the given section may now be obtained by the algebraic sum of the moment of inertia about the required axis.

Example 7.11. *An I-section is made up of three rectangles as shown in Fig. 7.15. Find the moment of inertia of the section about the horizontal axis passing through the centre of gravity of the section.*

Solution. First of all, let us find out centre of gravity of the section. As the section is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis. Split up the whole section into three rectangles 1, 2 and 3 as shown in Fig. 7.15, Let bottom face of the bottom flange be the axis of reference.

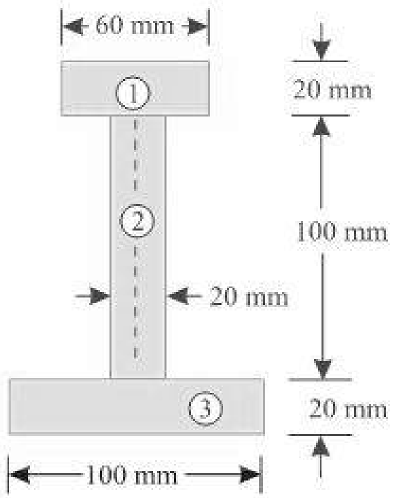


Fig. 7.15.

(*i*) *Rectangle* 1

$$a_1 = 60 \times 20 = 1200 \text{ mm}$$

and $$y_1 = 20 + 100 + \frac{20}{2} = 130 \text{ mm}$$

(*ii*) *Rectangle* 2

$$a_2 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_2 = 20 + \frac{100}{2} = 70 \text{ mm}$$

(*iii*) *Rectangle* 3

$$a_3 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_3 = \frac{20}{2} = 10 \text{ mm}$$

We know that the distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(1200 \times 130) + (2000 \times 70) + (2000 \times 10)}{1200 + 2000 + 2000} \text{ mm}$$

$$= 60.8 \text{ mm}$$

We know that moment of inertia of rectangle (1) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G1} = \frac{60 \times (20)^3}{12} = 40 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (1) and X-X axis,

$$h_1 = 130 - 60.8 = 69.2 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about X-X axis,

$$= I_{G1} + a_1 h_1^2 = (40 \times 10^3) + [1200 \times (69.2)^2] = 5786 \times 10^3 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G2} = \frac{20 \times (100)^3}{12} = 1666.7 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (2) and X-X axis,

$$h_2 = 70 - 60.8 = 9.2 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about X-X axis,

$$= I_{G2} + a_2 h_2^2 = (1666.7 \times 10^3) + [2000 \times (9.2)^2] = 1836 \times 10^3 \text{ mm}^4$$

Now moment of inertia of rectangle (3) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G3} = \frac{100 \times (20)^3}{12} = 66.7 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (3) and X-X axis,

$$h_3 = 60.8 - 10 = 50.8 \text{ mm}$$

∴ Moment of inertia of rectangle (3) about X-X axis,

$$= I_{G3} + a_3 h_3^2 = (66.7 \times 10^3) + [2000 \times (50.8)^2] = 5228 \times 10^3 \text{ mm}^4$$

Now moment of inertia of the whole section about X-X axis,

$$I_{XX} = (5786 \times 10^3) + (1836 \times 10^3) + (5228 \times 10^3) = 12\,850 \times 10^3 \text{ mm}^4$$ **Ans.**

Example 7.12. *Find the moment of inertia about the centroidal X-X and Y-Y axes of the angle section shown in Fig. 7.16.*

Solution. First of all, let us find the centre of gravity of the section. As the section is not symmetrical about any section, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the angle section. Split up the section into two rectangles (1) and (2) as shown in Fig. 7.16.

Moment of inertia about centroidal X-X axis

Let bottom face of the angle section be the axis of reference.

Rectangle (1)

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

Rectangle (2)

$$a_2 = (80 - 20) \times 20 = 1200 \text{ mm}^2$$

and $$y_2 = \frac{20}{2} = 10 \text{ mm}$$

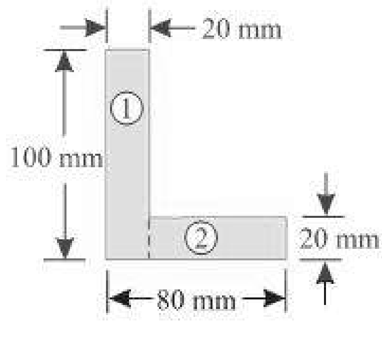


Fig. 7.16.

We know that distance between the centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35 \text{ mm}$$

We know that moment of inertia of rectangle (1) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G1} = \frac{20 \times (100)^3}{12} = 1.667 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangle (1) from *X-X* axis,

$$h_1 = 50 - 35 = 15 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about *X-X* axis

$$= I_{G1} + a h_1^2 = (1.667 \times 10^6) + [2000 \times (15)^2] = 2.117 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G2} = \frac{60 \times (20)^3}{12} \; 0.04 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangle (2) from *X-X* axis,

$$h_2 = 35 - 10 = 25 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about *X-X* axis

$$= I_{G2} + a h_2^2 = (0.04 \times 10^6) + [1200 \times (25)^2] = 0.79 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about *X-X* axis,

$$I_{XX} = (2.117 \times 10^6) + (0.79 \times 10^6) = 2.907 \times 10^6 \text{ mm}^4 \qquad \textbf{Ans.}$$

Moment of inertia about centroidal Y-Y axis

Let left face of the angle section be the axis of reference.

Rectangle (1)

$$a_1 = 2000 \text{ mm}^2 \qquad \text{...(As before)}$$

and $$x_1 = \frac{20}{2} = 10 \text{ mm}$$

Rectangle (2)

$$a_2 = 1200 \text{ mm}^2 \qquad \text{...(As before)}$$

and $$x_2 = 20 + \frac{60}{2} = 50 \text{ mm}$$

We know that distance between the centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \text{ mm}$$

We know that moment of inertia of rectangle (1) about an axis through its centre of gravity and parallel to *Y-Y* axis,

$$I_{G1} = \frac{100 \times (20)^3}{12} = 0.067 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangle (1) from *Y-Y* axis,

$$h_1 = 25 - 10 = 15 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about *Y-Y* axis

$$= I_{G1} + a_1 h_1^2 = (0.067 \times 10^6) + [2000 \times (15)^2] = 0.517 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to *Y-Y* axis,

$$I_{G2} = \frac{20 \times (60)^3}{12} = 0.36 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangle (2) from *Y-Y* axis,

$$h_2 = 50 - 25 = 25 \text{ mm},$$

∴ Moment of inertia of rectangle (2) about *Y-Y* axis

$$= I_{G2} + a_2 h_2^2 = 0.36 \times 10^6 + [1200 \times (25)^2] = 1.11 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about *Y-Y* axis,

$$I_{YY} = (0.517 \times 10^6) + (1.11 \times 10^6) = 1.627 \times 10^6 \text{ mm}^4 \qquad \textbf{Ans.}$$

Example 7.10. *Find the moment of inertia of a T-section with flange as 150 mm × 50 mm and web as 150 mm × 50 mm about X-X and Y-Y axes through the centre of gravity of the section.*

Solution. The given *T*-section is shown in Fig. 7.14.

First of all, let us find out centre of gravity of the section. As the section is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis. Split up the whole section into two rectangles *viz.*, 1 and 2 as shown in figure. Let bottom of the web be the axis of reference.

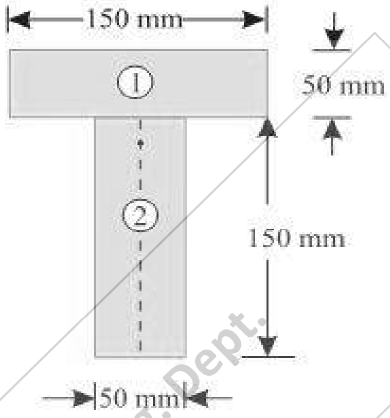


Fig. 7.14.

(*i*) *Rectangle* (1)

$$a_1 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_1 = 150 + \frac{50}{2} = 175 \text{ mm}$$

(*ii*) *Rectangle* (2)

$$a_2 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_2 = \frac{150}{2} = 75 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the web,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500} = 125 \text{ mm}$$

Moment of inertia about X-X axis

We also know that M.I. of rectangle (1) about an axis through its centre of gravity and parallel to *X-X* axis.

$$I_{G1} = \frac{150\,(50)^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of rectangle (1) and *X-X* axis,

$$h_1 = 175 - 125 = 50 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about *X-X* axis

$$I_{G1} + a_1 h_1^2 = (1.5625 \times 10^6) + [7500 \times (50)^2] = 20.3125 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G2} = \frac{50\,(150)^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of rectangle (2) and *X-X* axis,

$$h_2 = 125 - 75 = 50 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about *X-X* axis

$$= I_{G2} + a_2 h_2^2 = (14.0625 \times 10^6) + [7500 \times (50)^2] = 32.8125 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about *X-X* axis,

$$I_{XX} = (20.3125 \times 10^6) + (32.8125 \times 10^6) = 53.125 \times 10^6 \text{ mm}^4 \quad \textbf{Ans.}$$

Moment of inertia about Y-Y axis

We know that M.I. of rectangle (1) about *Y-Y* axis

$$= \frac{50\,(150)^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

and moment of inertia of rectangle (2) about *Y-Y* axis,

$$= \frac{150\,(50)^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about *Y-Y* axis,

$$I_{YY} = (14.0625 \times 10^6) + (1.5625 \times 10^6) = 15.625 \times 10^6 \text{ mm}^4 \quad \textbf{Ans.}$$

Chapter-05: Simple Machines

5.1 Machine

- The device that transmits or modifies force or motion.
- A thing made for a particular purpose.
- An apparatus consisting of interrelated parts with separate functions, used in the performance of some kind of work.
- A machine uses power to apply forces and control movement to perform an intended action.
- The input of a machine is the work done on the machine.
- The output of a machine is the actual work done by the machine.

Simple Machine

A simple machine may be defined as a device, which enables us to do some useful work at some point or to overcome some resistance, when an effort or force is applied to it, at some other convenient point.

Compound Machine

A compound machine may be defined as a device, consisting of a number of simple machines, which enables us to do some useful work at a faster speed or with a much less effort as compared to a simple machine.

Lifting Machine

- It is a device, which enables us to lift a heavy load (W) by applying a comparatively smaller effort (P).
- In a lifting machine, input is measured by the product of effort and the distance through which it has moved.
- In a lifting machine, output is measured by the product of the weight lifted and the distance through which it has been lifted..

Efficiency of a Machine

It is the ratio of output to the input of a machine and is generally expressed as a percentage. Mathematically, efficiency,

$$\eta = \frac{\text{Output}}{\text{Input}} \times 100$$

Ideal Machine

If the efficiency of a machine is 100% i.e., if the output is equal to the input, the machine is called as a perfect or an ideal machine.

Mechanical Advantage

The mechanical advantage (M.A.) is the ratio of weight lifted (W) to the effort applied (P) and is always expressed in pure number. Mathematically, mechanical advantage,

$$\text{M.A.} = W/P.$$

Velocity Ratio

The velocity ratio (V.R.) is the ratio of distance moved by the effort (y) to the distance moved by the load (x) and is always expressed in pure number. Mathematically, velocity ratio,

$$\text{V.R.} = y/x,$$

Relation between Efficiency, Mechanical Advantage and Velocity Ratio of a Lifting Machine

Let W = Load lifted by the machine,

P = Effort required to lift the load,

Y = Distance moved by the effort, in lifting the load, and

x = Distance moved by the load.

We know that M.A. = W/P and V.R. = y/x

We also know that input of a machine

= Effort applied × Distance through which the effort has moved

= P × y ...(i)

and output of a machine

= Load lifted × Distance through which the load has been lifted

= W × x

$$\therefore \quad \text{Efficiency,} \qquad \eta = \frac{\text{Output}}{\text{Input}} = \frac{W \times x}{P \times y} = \frac{W/P}{y/x} = \frac{\text{M.A.}}{\text{V.R.}}$$

Example 10.1. *In a certain weight lifting machine, a weight of 1 kN is lifted by an effort of 25 N. While the weight moves up by 100 mm, the point of application of effort moves by 8 m. Find mechanical advantage, velocity ratio and efficiency of the machine.*

Solution. Given: Weight (W) = 1 kN = 1000 N ; Effort (P) = 25 N ; Distance through which the weight is moved (x) = 100 mm = 0.1 m and distance through which effort is moved (y) = 8 m.

Mechanical advantage of the machine.

We know that mechanical advantage of the machine

$$\text{M.A.} = \frac{W}{P} = \frac{1000}{25} = 40 \qquad \textbf{Ans.}$$

Velocity ratio of the machine

We know that velocity ratio of the machine

$$\text{V.R.} = \frac{y}{x} = \frac{8}{0.1} = 80 \qquad \textbf{Ans.}$$

Efficiency of the machine

We also know that efficiency of the machine,

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{40}{80} = 0.5 = 50\% \qquad \textbf{Ans.}$$

Reversibility of a Machine

When a machine is also capable of doing some work in the reversed direction, after the effort is removed, it is called a reversible machine and its action is known as reversibility of the machine.

Relation between Efficiency, Mechanical Advantage and Velocity Ratio of a Lifting Machine

Let W = Load lifted by the machine,

P = Effort required to lift the load,

y = Distance moved by the effort, and

x = Distance moved by the load.

We know that,

input of the machine $= P \times y$ ...(i)

and output of the machine $= W \times x$ ...(ii)

We also know that machine friction

$= \text{Input} - \text{Output} = (P \times y) - (W \times x)$ ...(iii)

A little consideration will show that in a reversible machine, the output of the machine should be more than the machine friction, when the effort (P) is zero. i.e.,

$$W \times x > P \times y - W \times x$$

or $$2\,W \times x > P \times y$$

or $$(W \times x) / (P \times y) > 1/2$$

or $$(W/P) / (y/x) > 1/2$$

or $$M.A./V.R. > 1/2$$

…(W/P = M.A. and y/x = V.R.)

$$\eta > 1/2 = 0.5 = 50\%$$

Hence the condition for a machine, to be reversible, is that its efficiency should be more than 50%.

consideration

Self-Locking Machine

- Sometimes, a machine is not capable of doing any work in the reversed direction, after the effort is removed. Such a machine is called a non-reversible or self-locking machine.
- The condition for a machine to be non-reversible or self-locking is that its efficiency should not be more than 50%.

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Example 10.3. *In a lifting machine, whose velocity ratio is 50, an effort of 100 N is required to lift a load of 4 kN. Is the machine reversible ? If so, what effort should be applied, so that the machine is at the point of reversing ?*

Solution. Given: Velocity ratio (V.R.) = 50 ; Effort (P) = 100 N and load (W) = 4 kN = 4000 N.

Reversibility of the machine

We know that $\text{M.A.} = \dfrac{W}{P} = \dfrac{4000}{100} = 40$

and efficiency, $\eta = \dfrac{\text{M.A.}}{\text{V.R.}} = \dfrac{40}{50} = 0.8 = 80\%$

Since efficiency of the machine is more than 50%, therefore the machine is reversible. **Ans.**

Effort to be applied

A little consideration will show that the machine will be at the point of reversing, when its efficiency is 50% or 0.5.

Let P_1 = Effort required to lift a load of 4000 N when the machine is at the point of reversing.

We know that $\text{M.A.} = \dfrac{W}{P_1} = \dfrac{4000}{P_1} = 4000/P_1$

and efficiency, $0.5 = \dfrac{\text{M.A.}}{\text{V.R.}} = \dfrac{4000/P_1}{50} = \dfrac{80}{P_1}$

$\therefore \quad P_1 = \dfrac{80}{0.5} = 160 \text{ N}$ **Ans.**

Example 10.2. *A certain weight lifting machine of velocity ratio 30 can lift a load of 1500 N with the help of 125 N effort. Determine if the machine is reversible.*

Solution. Given: Velocity ratio (V.R.) = 30; Load (W) = 1500 N and effort (P) = 125 N.

We know that $\text{M.A.} = \dfrac{W}{P} = \dfrac{1500}{125} = 12$

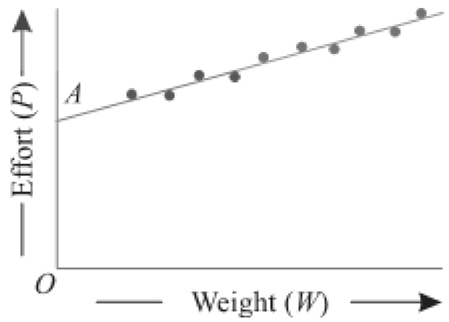
and efficiency, $\eta = \dfrac{\text{M.A.}}{\text{V.R.}} = \dfrac{12}{30} = 0.4 = 40\%$

Since efficiency of the machine is less than 50%, therefore the machine is non-reversible. **Ans.**

Law of a Machine

may be defined as relationship between the effort applied and the load lifted. Thus for any machine, if we record the various efforts required to raise the corresponding loads, and plot a graph between effort and load, we shall get a straight line.

The intercept OA represents the amount of friction offered by the machine. Or in other words, this is the effort required by the machine to overcome the friction, before it can lift any load.



Mathematically, the law of a lifting machine is given by the relation:

$$P = mW + C$$

where P = Effort applied to lift the load, m = A constant (called coefficient of friction) which is equal to the slope of the line AB,

W = Load lifted, and

C = Another constant, which represents the machine friction, (i.e. OA).

Example 10.5. *What load can be lifted by an effort of 120 N, if the velocity ratio is 18 and efficiency of the machine at this load is 60% ?*

Determine the law of the machine, if it is observed that an effort of 200 N is required to lift a load of 2600 N and find the effort required to run the machine at a load of 3.5 kN.

Solution. Given: Effort $(P) = 120$ N ; Velocity ratio (V.R.) = 18 and efficiency $(\eta) = 60\% = 0.6$.

Load lifted by the machine.

Let W = Load lifted by the machine.

We know that $\text{M.A.} = \frac{W}{P} = \frac{W}{120} = W/120$

and efficiency, $0.6 = \frac{\text{M.A.}}{\text{V.R.}} = \frac{W/120}{18} = \frac{W}{2160}$

$\therefore \quad W = 0.6 \times 2160 = 1296 \text{ N}$ **Ans.**

Law of the machine

In the second case, $P = 200$ N and $W = 2600$ N

Substituting the two values of P and W in the law of the machine, *i.e.*, $P = m\,W + C$,

$$120 = m \times 1296 + C \quad \text{...(i)}$$

and $$200 = m \times 2600 + C \quad \text{...(ii)}$$

Subtracting equation (*i*) from (*ii*),

$$80 = 1304\,m \quad \text{or} \quad m = \frac{80}{1304} = 0.06$$

and now substituting the value of m in equation (*ii*)

$$200 = (0.06 \times 2600) + C = 156 + C$$

$$C = 200 - 156 = 44$$

Now substituting the value of $m = 0.06$ and $C = 44$ in the law of the machine,

$$P = 0.06\,W + 44$$ **Ans.**

Effort required to run the machine at a load of 3.5 kN.

Substituting the value of W = 3.5 kN or 3500 N in the law of machine,

$$P = (0.06 \times 3500) + 44 = 254 \text{ N}$$ **Ans.**

Example 10.6. *In a lifting machine, an effort of 40 N raised a load of 1 kN. If efficiency of the machine is 0.5, what is its velocity ratio ? If on this machine, an effort of 74 N raised a load of 2 kN, what is now the efficiency ? What will be the effort required to raise a load of 5 kN ?*

Solution. Given: When Effort $(P) = 40$ N; Load $(W) = 1$ kN = 1000 N; Efficiency $(\eta) = 0.5$; When effort $(P) = 74$ N and load $(W) = 2$ kN = 2000 N.

Velocity ratio when efficiency is 0.5.

We know that $\text{M.A.} = \frac{W}{P} = \frac{1000}{40} = 25$

and efficiency $0.5 = \frac{\text{M.A.}}{\text{V.R.}} = \frac{25}{\text{V.R.}}$

$\therefore \quad \text{V.R.} = \frac{25}{0.5} = 50$ **Ans.**

Efficiency when P is 74 N and W is 2000 N

We know that $\text{M.A.} = \frac{W}{P} = \frac{2000}{74} = 27$

and efficiency $\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{27}{50} = 0.54 = 54\%$ **Ans.**

Effort required to raise a load of 5 kN or 5000 N

Substituting the two values of P and W in the law of the machine, *i.e.* $P = mW + C$

$$40 = m \times 1000 + C \quad ...(i)$$

and

$$74 = m \times 2000 + C \quad ...(ii)$$

Subtracting equation (*i*) from (*ii*),

$$34 = 1000\, m \quad \text{or} \quad m = \frac{34}{1000} = 0.034$$

and now substituting this value of m in equation (*i*),

$$40 = (0.034 \times 1000) + C = 34 + C$$

$$\therefore \quad C = 40 - 34 = 6$$

Substituting these values of $m = 0.034$ and $C = 6$ in the law of machine,

$$P = 0.034\, W + 6 \quad ...(iii)$$

$\therefore$ Effort required to raise a load of 5000 N,

$$P = (0.034 \times 5000) + 6 = 176 \text{ N}$$ **Ans.**

Example 10.7. *What load will be lifted by an effort of 12 N, if the velocity ratio is 18 and efficiency of the machine at this load is 60 % ?*

If the machine has a constant friction resistance, determine the law of the machine and find the effort required to run this machine at (i) no load, and (ii) a load of 900 N.

Solution. Given: Effort (P) = 12 N ; Velocity ratio (V.R.) = 18 and efficiency (η) = 60 % = 0.6.

Load lifted by the machine.

Let W = Load lifted by the machine,

We know that $\text{M.A.} = \frac{W}{P} = \frac{W}{12} = W/12$

and efficiency, $0.6 = \frac{\text{M.A.}}{\text{V.R.}} = \frac{W/12}{18} = \frac{W}{216}$

$$\therefore \quad W = 0.6 \times 216 = 129.6 \text{ N}$$ **Ans.**

Law of the machine

We know that effort lost in friction,

$$F_{(\text{effort})} = P - \frac{W}{\text{V.R.}} = 12 - \frac{129.6}{18} = 4.8 \text{ N}$$

Since the frictional resistance is constant, therefore 4.8 N is the amount of friction offered by the machine. Now substituting the values of $P = 12$ and $C = 4.8$ in the law of the machine.

$$12 = m \times 129.6 + 4.8 \quad ...(\because P = mW + C)$$

or

$$m = \frac{12 - 4.8}{129.6} = \frac{1}{18}$$

$\therefore$ Law of the machine will be given by the equation,

$$P = \frac{1}{18} W + 4.8$$ **Ans.**

Effort required to run the machine at no load

Substituting the value of $W = 0$ in the law of the machine (for no load condition),

$$P = 4.8 \text{ N}$$ **Ans.**

Effort required to run the machine at a load of 900 N

Substituting the value of $W = 900$ N in the law of machine,

$$P = \frac{1}{18} \times 900 + 4.8 = 54.8 \text{ N}$$ **Ans.**

Maximum Mechanical Advantage of a Lifting Machine

We know that mechanical advantage of a lifting machine,

$$M.A. = \frac{W}{P}$$

For maximum mechanical advantage, substituting the value of $P = mW + C$ in the above equation,

$$\text{Max. M.A.} = \frac{W}{mW + C} = \frac{1}{m + \frac{C}{W}} = \frac{1}{m} \quad \ldots \left(\text{Neglecting } \frac{C}{W}\right)$$

Maximum Efficiency of a Lifting Machine

We know that efficiency of a lifting machine,

$$\eta = \frac{\text{Mechanical advantage}}{\text{Velocity ratio}} = \frac{\frac{W}{P}}{V.R.} = \frac{W}{P \times V.R.}$$

For *maximum efficiency, substituting the value of $P = mW + C$ in the above equation,

$$\text{Max. } \eta = \frac{W}{(mW + C) \times V.R} = \frac{1}{\left(m + \frac{C}{W}\right) \times V.R.} = \frac{1}{m \times V.R.} \quad \ldots \left(\text{Neglecting } \frac{C}{W}\right)$$

Example 10.8. *The law of a machine is given by the relation :*

$$P = 0.04\ W + 7.5$$

where (P) is the effort required to lift a load (W), both expressed in newtons. What is the mechanical advantage and efficiency of the machine, when the load is 2 kN and velocity ratio is 40 ? What is the maximum efficiency of the machine ?

If (F) is the effort lost in friction, find the relation between F and W. Also find the value of F, when W is 2 kN.

Solution. Given: Law of machine $P = 0.04\ W + 7.5$; Load $(W) = 2$ kN $= 2000$ N and velocity ratio (V.R.) = 40.

Mechanical advantage

Substituting the value of W in the law of the machine,

$$P = mW + C = 0.04 \times 2000 + 7.5 = 87.5 \text{ N}$$

$$\therefore \quad M.A. = \frac{W}{P} = \frac{2000}{87.5} = 22.9 \quad \textbf{Ans.}$$

Efficiency of the machine

We know that $\eta = \frac{M.A.}{V.R.} = \frac{22.9}{40} = 0.5725 = 57.25\%$ **Ans.**

Maximum efficiency of the machine

We know that *maximum efficiency of the machine,

$$\text{Max.}\eta = \frac{1}{m \times V.R.} = \frac{1}{0.04 \times 40} = 0.625 = 62.5\% \quad \textbf{Ans.}$$

Relation between F and W

We know that effort lost in friction,

$$F_{(effort)} = P - \frac{W}{V.R.} = (0.04\ W + 7.5) - \frac{W}{V.R.}$$

$$= W\left(0.04 - \frac{1}{V.R.}\right) + 7.5 = W\left(0.04 - \frac{1}{40}\right) + 7.5$$

$$= W(0.04 - 0.025) + 7.5 = 0.015\ W + 7.5 \quad \textbf{Ans.}$$

Value of F when W is 2 kN

Substituting the value of W equal to 2 kN or 2000 N in the above equation,

$$F = (0.015 \times 2000) + 7.5 = 37.5 \text{ N} \quad \textbf{Ans.}$$

Example 10.9. *The law of a certain lifting machine is :*

$$P = \frac{W}{50} + 8$$

The velocity ratio of the machine is 100. Find the maximum possible mechanical advantage and the maximum possible efficiency of the machine. Determine the effort required to overcome the machine friction, while lifting a load of 600 N. Also calculate the efficiency of the machine at this load.

Solution. Given: Law of lifting machine $P = \frac{W}{50} + 8 = 0.02W + 8$; Velocity ratio (V.R.) = 100 and load (W) = 600 N.

Maximum possible mechanical advantage

Comparing the given law of the machine with the standard relation for the law of the machine (*i.e.* $P = mW + C$) we find that in the given law of the machine, $m = 0.02$. We know that maximum possible mechanical advantage

$$\text{Max M.A.} = \frac{1}{m} = \frac{1}{0.02} = 50 \quad \textbf{Ans.}$$

Maximum possible efficiency

We know that maximum possible efficiency

$$= \frac{1}{m \times \text{V.R.}} = \frac{1}{0.02 \times 100} = \frac{1}{2} = 0.5 = 50\% \quad \textbf{Ans.}$$

Effort required to overcome the machine friction

We know that effort required to lift a load of 600 N

$$P = mW + 8 = (0.02 \times 600) + 8 = 20 \text{ N}$$

and effort required to overcome the machine friction, while lifting a load of 600 N,

$$F_{(effort)} = P - \frac{W}{\text{V.R.}} = 20 - \frac{600}{100} = 14 \text{ N} \quad \textbf{Ans.}$$

Efficiency of the machine

We know that mechanical advantage of the machine while lifting a load of 600 N.

$$\text{M.A} = \frac{W}{P} = \frac{600}{20} = 30$$

and efficiency, $$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{30}{100} = 0.3 = 30\% \quad \textbf{Ans.}$$

5.2 Study of Simple Machines

Simple Axle and Wheel

In Figure a simple wheel and axle, in which the wheel A and axle B are keyed to the same shaft. The shaft is mounted on ball bearings, in order to reduce the frictional resistance to a minimum. A string is wound round the axle B, which carries the load to be lifted. A second string is wound round the wheel A in the opposite direction to that of the string on B.

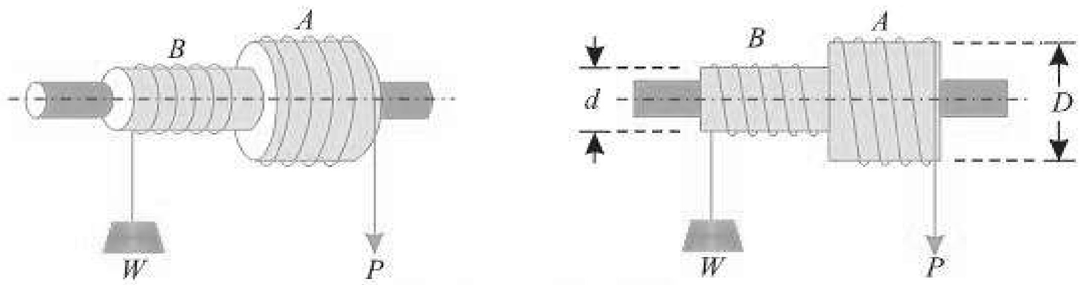


Fig. 11.1. Simple wheel and axle.

Let D = Diameter of effort wheel,

d = Diameter of the load axle,

W = Load lifted, and

P = Effort applied to lift the load.

One end of the string is fixed to the wheel, while the other is free and the effort is applied to this end. Since the two strings are wound in opposite directions, therefore a downward motion of the effort (P) will raise the load (W).

Since the wheel as well as the axle is keyed to the same shaft, therefore when the wheel rotates through one revolution, the axle will also rotate through one revolution. We know that displacement of the effort in one revolution of effort wheel A,

$$= \pi D$$

and displacement of the load in one revolution

$$= \pi d$$

$$\therefore \quad \text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{\pi D}{\pi d} = \frac{D}{d}$$

Now $$\text{M.A.} = \frac{\text{Load lifted}}{\text{Effort applied}} = \frac{W}{P}$$ ..as usual

and efficiency $$\eta = \frac{\text{M.A}}{\text{V.R.}}$$ ...as usual

Example 11.2. *A drum weighing 60 N and holding 420N of water is to be raised from a well by means of wheel and axle. The axle is 100 mm diameter and the wheel is 500 mm diameter. If a force of 120 N has to be applied to the wheel, find (i) mechanical advantage, (ii) velocity ratio and (iii) efficiency of the machine.*

Solution. Given: Total load to be lifted (W) = 60 + 420 = 480 N; Diameter of the load axle (d) = 100 mm; Diameter of effort wheel (D) = 500 mm and effort (P) = 120 N.

Mechanical advantage

We know that mechanical advantage

$$\text{M.A.} = \frac{W}{P} = \frac{480}{120} = 4 \quad \textbf{Ans.}$$

Velocity ratio

We know that velocity ratio

$$\text{V.R.} = \frac{D}{d} = \frac{500}{100} = 5 \quad \textbf{Ans.} \qquad ...(ii)$$

Efficiency of the machine

We also know that efficiency of the machine,

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{4}{5} = 0.8 = 80\% \quad \textbf{Ans.}$$

Note : If we consider weight of the water only (*i.e.*, neglecting weight of the drum) then

$$\text{M.A.} = \frac{420}{120} = 3.5 \quad \textbf{Ans.}$$

and efficiency $$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{3.5}{5} = 0.7 = 70\% \quad \textbf{Ans.}$$

Example 11.1. *A simple wheel and axle has wheel and axle of diameters of 300 mm and 30 mm respectively. What is the efficiency of the machine, if it can lift a load of 900 N by an effort of 100 N.*

Solution. Given: Diameter of wheel (D) = 300 mm; Diameter of axle (d) = 30 mm; Load lifted by the machine (W) = 900 N and effort applied to lift the load (P) = 100 N

We know that velocity ratio of the simple wheel and axle,

$$\text{V.R.} = \frac{D}{d} = \frac{300}{30} = 10$$

and mechanical advantage
$$\text{M.A.} = \frac{W}{P} = \frac{900}{100} = 9$$

$$\therefore \quad \text{Efficiency, } \eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{9}{10} = 0.9 = 90\% \qquad \textbf{Ans.}$$

Worm and Worm Wheel

It consists of a square threaded screw, S (known as worm) and a toothed wheel (known as worm wheel) geared with each other. A wheel A is attached to the worm, over which passes a rope as shown in the figure. A load drum is securely mounted on the worm wheel.

Let D = Diameter of the effort wheel,

r = Radius of the load drum

W = Load lifted,

P = Effort applied to lift the load, and

T = No. of teeth on the worm wheel.

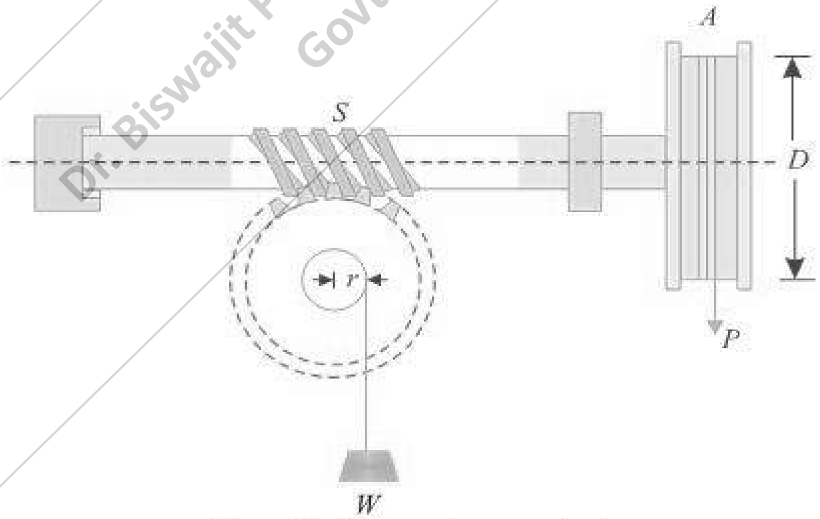


Fig. 11.5. Worm and worm wheel

We know that distance moved by the effort in one revolution of the wheel (or handle)

$= \pi D$

If the worm is single-threaded (i.e., for one revolution of the wheel A, the screw S pushes the worm wheel through one teeth), then the load drum will move through

$= (1 / T)$ revolution

and distance, through which the load will move $= 2\pi r / T$

$$\therefore \quad \text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$$

$$= \frac{\pi D}{\frac{2\pi r}{T}} = \frac{DT}{2r} \quad \text{...(iii)}$$

Now $$\text{M.A.} = \frac{W}{P} \quad \text{...as usual}$$

and efficiency, $$\eta = \frac{\text{M.A.}}{\text{V.R.}} \quad \text{...as usual}$$

Notes : 1. If the worm is double-threaded *i.e.*, for one revolution of wheel *A*, the screw *S* pushes the worm wheel through two teeths, then

$$\text{V.R.} = \frac{DT}{2 \times 2r} = \frac{DT}{4r}$$

2. In general, if the worm is *n* threaded, then

$$\text{V.R.} = \frac{DT}{2nr}$$

Example 11.9. *A worm and worm wheel with 40 teeth on the worm wheel has effort wheel of 300 mm diameter and load drum of 100 mm diameter. Find the efficiency of the machine, if it can lift a load of 1800 N with an effort of 24 N.*

Solution. Given: No. of teeth on the worm wheel (T) = 40 ; Diameter of effort wheel = 300 mm Diameter of load drum = 100 mm or radius (r) = 50 mm; Load lifted (W) 1800 N and effort (P) = 24 N.

We know that velocity ratio of worm and worm wheel,

$$\text{V.R.} = \frac{DT}{2r} = \frac{300 \times 40}{2 \times 50} = 120$$

and $$\text{M.A.} = \frac{W}{P} = \frac{1800}{24} = 75$$

$\therefore$ Efficiency, $$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{75}{120} = 0.625 = 62.5\% \quad \textbf{Ans.}$$

Example 11.10. *In a double threaded worm and worm wheel, the number of teeth on the worm wheel is 60. The diameter of the effort wheel is 250 mm and that of the load drum is 100 mm. Calculate the velocity ratio. If the efficiency of the machine is 50%, determine the effort required to lift a load of 300 N.*

Solution. Given : No. of threads (n) = 2; No. of teeth on the worm wheel (T) = 60; Diameter of effort wheel = 250 mm; Diameter of load drum = 100 mm or radius (r) = 50 mm; Efficiency (η) = 50% = 0.5 and load to be lifted (W) = 300 N.

Velocity ratio of the machine

We know that velocity ratio of a worm and worm wheel,

$$\text{V.R.} = \frac{DT}{2nr} = \frac{250 \times 60}{2 \times 2 \times 50} = 75 \quad \textbf{Ans.}$$

Effort required to lift the load

Let P = Effort required to lift the load.

We also know that mechanical advantage,

$$\text{M.A.} = \frac{W}{P} = \frac{300}{P}$$

and efficiency, $$0.5 = \frac{\text{M.A.}}{\text{V.R.}} = \frac{\frac{300}{P}}{75} = \frac{4}{P}$$

or $$P = \frac{4}{0.5} = 8 \text{ N} \quad \textbf{Ans.}$$

Single Purchase Crab Winch

In single purchase crab winch, a rope is fixed to the drum and is wound a few turns round it. The free end of the rope carries the load **W**. A toothed wheel **A** is rigidly mounted on the load drum. Another toothed wheel **B**, called pinion, is geared with the toothed wheel **A**.

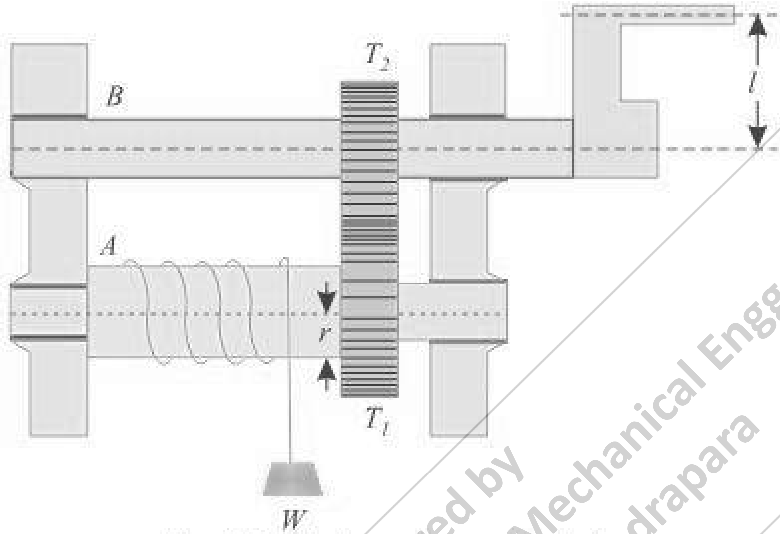


Fig. 11.7. Single purchase crab winch.

Let T_1 = No. of teeth on the main gear (or spur wheel) A,

T_2 = No. of teeth on the pinion B,

l = Length of the handle,

r = Radius of the load drum,

W = Load lifted, and

P = Effort applied to lift the load.

We know that distance moved by the effort in one revolution of the handle,

$$= 2\pi l \qquad \text{...(i)}$$

∴ No. of revolutions made by the pinion B

$$= 1$$

and no. of revolutions made by the wheel A

$$= T_2 / T_1$$

∴ No. of revolutions made by the load drum

$$= T_2 / T_1$$

and distance moved by the load = $2\pi r \times (T_2 / T_1)$ ...(ii)

∴ $$\text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{2\pi l}{2\pi r \times \frac{T_2}{T_1}} = \frac{l}{r} \times \frac{T_1}{T_2} \qquad \text{...(iii)}$$

Now $$\text{M.A.} = \frac{W}{P} \qquad \text{...as usual}$$

and efficiency, $$\eta = \frac{\text{M.A.}}{\text{V.R.}} \qquad \text{...as usual}$$

Example 11.13. *In a single purchase crab winch, the number of teeth on pinion is 25 and that on the spur wheel 100. Radii of the drum and handle are 50 mm and 300 mm respectively. Find the efficiency of the machine and the effect of friction, if an effort of 20 N can lift a load of 300 N.*

Solution. Given: No. of teeth on pinion (T_2) = 25; No. of teeth on the spur wheel (T_1) = 100; Radius of daum (r) = 50 mm; Radius of the handle or length of the handle (l) = 300 mm; Effort (P) = 20 N and load lifted (W) = 300 N.

Efficiency of the machine

We know that velocity ratio

$$\text{V.R.} = \frac{l}{r} \times \frac{T_1}{T_2} = \frac{300}{50} \times \frac{100}{25} = 24$$

and

$$\text{M.A.} = \frac{W}{P} = \frac{300}{20} = 15$$

∴ Efficiency,

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{15}{24} = 0.625 = 62.5\% \qquad \textbf{Ans.}$$

Effect of friction

We know that effect of friction in terms of load,

$$F_{(\text{load})} = (P \times \text{V.R.}) - W = (20 \times 24) - 300 = 180 \text{ N}$$

and effect of friction in terms of effort,

$$F_{(\text{effort})} = P - \frac{W}{\text{V.R.}} = 20 - \frac{300}{24} = 7.5 \text{ N}$$

It means that if the machine would have been ideal (*i.e.* without friction) then it could lift an extra load of 180 N with the same effort of 20 N. Or it could have required 7.5 N less force to lift the same load of 300 N. **Ans.**

Example 11.14. *A single purchase crab winch, has the following details:*

Length of lever	*= 700 mm*
Number of pinion teeth	*= 12*
Number of spur gear teeth	*= 96*
Diameter of load axle	*= 200 mm*

It is observed that an effort of 60 N can lift a load of 1800 N and an effort of 120 N can lift a load of 3960 N.

What is the law of the machine ? Also find efficiency of the machine in both the cases.

Solution. Given: Length of lever (l) = 700 *mm*; No. of pinion teeth (T_2) = 12; No. of spur geer teeth (T_1) = 96 and dia of load axle = 200 mm or radius (r) = 200/2 = 100 mm.

(*i*) *Law of the machine*

When P_1 = 60 N, W_1 = 1800 N and when P_2 = 120 N, W_2 = 3960 N.

Substituting the values of P and W in the law of the machine *i.e.*, $P = mW + C$

$$60 = (m \times 1800) + C \qquad \ldots(i)$$

and

$$120 = (m \times 3960) + C \qquad \ldots(ii)$$

Subtracting equation (*i*) from equation (*ii*)

$$60 = m \times 2160$$

or

$$m = \frac{60}{2160} = \frac{1}{36}$$

Now substituting this value of m in equation (*i*),

$$60 = \left(\frac{1}{36} \times 1800\right) + C = 50 + C$$

∴

$$C = 60 - 50 = 10$$

and now substituting the value of m = 1/36 and C = 10 in the law of machine,

$$P = \frac{1}{36} W + 10 \qquad \textbf{Ans.}$$

(ii) *Efficiencies of the machine in both the cases*

We know that velocity ratio

$$V.R. = \frac{l}{r} \times \frac{T_1}{T_2} = \frac{700}{100} \times \frac{96}{12} = 56$$

and mechanical advantage in the first case

$$M.A. = \frac{W_1}{P_1} = \frac{1800}{60} = 30$$

$\therefore$ Efficiency $\eta_1 = \frac{M.A}{V.R.} = \frac{30}{56} = 0.536 = 53.6\%$ **Ans.**

Similarly, mechanical advangate in the second case,

$$M.A. = \frac{W_2}{P_2} = \frac{3960}{120} = 33$$

$\therefore$ Efficiency $\eta_2 = \frac{M.A.}{V.R.} = \frac{33}{56} = 0.589 = 58.9\%$ **Ans.**

Double Purchase Crab Winch

In a double purchase crab winch the velocity ratio is intensified with the help of one more spur wheel and a pinion. There are two spur wheels of teeth T_1 and T_2 and T_3 as well as two pinions of teeth T_2 and T_4.

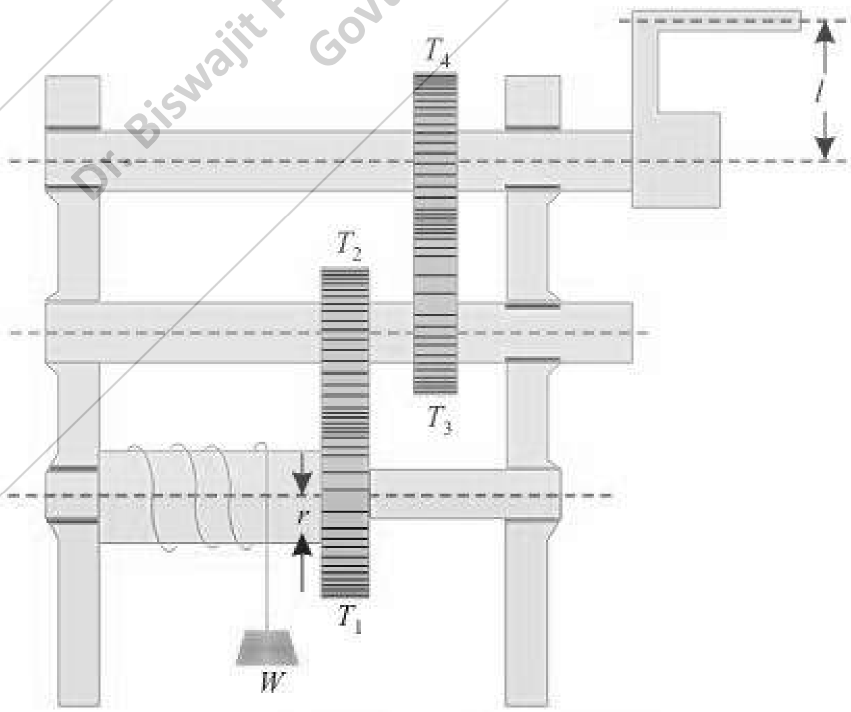


Fig. 11.8. Double purchase crab winch.

The arrangement of spur wheels and pinions are such that the spur wheel with T_1 gears with the pinion of teeth T_2. Similarly, the spur wheel with teeth T_3 gears with the pinion of the teeth T_4, The effort is applied to a handle.

Let T_1 and T_3 = No. of teeth of spur wheels,

T_2 and T_4 = No. of teeth on the pinion B,

l = Length of the handle,

r = Radius of the load drum,

W = Load lifted, and

P = Effort applied to lift the load, at the end of the handle.

We know that distance moved by the effort in one revolution of the handle,

$$= 2\pi l \qquad \text{...(i)}$$

∴ No. of revolutions made by the pinion 4

$$= l$$

and no. of revolutions made by the wheel 3

$$= T_4 / T_3$$

∴ No. of revolutions made by the pinion 2

$$= T_4 / T_3$$

and no. of revolutions made by the wheel 1

$$= (T_2 / T_1) \times (T_4 / T_3)$$

∴ distance moved by the load $= 2\pi r \times (T_2 / T_1) \times (T_4 / T_3)$ ...(ii)

$$\therefore \quad \text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$$

$$= \frac{2\pi l}{2\pi r \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}} = \frac{l}{r}\left(\frac{T_1}{T_2} \times \frac{T_3}{T_4}\right)$$

Now $\text{M.A.} = \frac{W}{P}$ ...as usual

and efficiency, $\eta = \frac{\text{M.A.}}{\text{V.R.}}$ ...as usual

Example 11.15. *In a double purchase crab winch*, teeth of pinions are *20 and 25 and that of spur wheels are 50 and 60. Length of the handle is 0.5 metre and radius of the load drum is 0.25 metre. If efficiency of the machine is 60%, find the effort required to lift a load of 720 N.*

Solution. Given: No. of teeth of pinion (T_2) = 20 and (T_4) = 25; No. of teeth of spur wheel (T_1) = 501 and (T_3) = 60; Length of the handle (l) = 0.5 m; Radius of the load drum (r) = 0.25 m; Efficiency (η) = 60% = 0.6 and load to be lifted (W) = 720 N.

Let P = Effort required in newton to lift the load.

We know that velocity ratio

$$\text{V.R.} = \frac{l}{r}\left(\frac{T_1}{T_2} \times \frac{T_3}{T_4}\right) = \frac{0.5}{0.25}\left(\frac{50}{20} \times \frac{60}{25}\right) = 12$$

and $$\text{M.A.} = \frac{W}{P} = \frac{720}{P}$$

∴ Efficiency $$0.6 = \frac{\text{M.A.}}{\text{V.R.}} = \frac{\frac{720}{P}}{12} = \frac{60}{P}$$

or $$P = \frac{60}{0.6} = 100 \text{ N}$$ **Ans.**

Example 11.16. *A double purchase crab used in a laboratory has the following dimensions :*

Diameter of load drum = *160 mm*
Length of handle = *360 mm*
No. of teeth on pinions = *20* and *30*
No. of teeth on spur wheels = *75* and *90*

When tested, it was found that an effort of 90 N was required to lift a load of 1800 N and an effort of 135 N was required to lift a load of 3150 N. Determine :

(*a*) *Law of the machine,*
(*b*) *Probable effort to lift a load of 4500 N,*
(*c*) *Efficiency of the machine in the above case,*
(*d*) *Maximum efficiency of the machine.*

Solution. Given: Dia of load drum = 160 mm or radius (r) = 100/2 = 80 mm; Length of handle (l) = 360 mm; No. of teeth on pinions (T_2) = 20 and (T_4) = 30 and no. of teeth on spus wheels (T_1) = 75 and (T_3) = 90.

When P = 90 N, W = 1800 N when P = 135 N, W = 3150 N

(*a*) *Law of the machine,*

Substituting the values of P and W in the law of the machine, *i.e.,* $P = m\,W + C$

$$90 = (m \times 1800) + C \qquad ...(i)$$

and

$$135 = (m \times 3150) + C \qquad ...(ii)$$

Subtracting equation (*i*) from equation (*ii*),

$$45 = m \times 1350$$

or

$$m = \frac{45}{1350} = \frac{1}{30}$$

Now substituting this value of m in equation (*i*),

$$90 = \frac{1}{30} \times 1800 + C = 60 + C$$

$$\therefore \quad C = 90 - 60 = 30$$

and now substituting the value fo m and C in the law of the machine,

$$P = \frac{1}{30} W + 30 \qquad \textbf{Ans.}$$

(*b*) *Effort to lift a load of* 4500 *N*

Substituting the value of W equal to 4500 N in the law of the machine,

$$P = \left(\frac{1}{30} \times 4500\right) + 30 = 180 \text{ N} \qquad \textbf{Ans.}$$

(*c*) *Efficiency of the machine in the above case*

We know that velocity ratio

$$\text{V.R.} = \frac{l}{r}\left(\frac{T_1 \times T_3}{T_2 \times T_4}\right) = \frac{360}{80}\left(\frac{75}{20} \times \frac{90}{30}\right) = 50.6$$

and

$$\text{M.A.} = \frac{W}{P} = \frac{4500}{180} = 25$$

$\therefore$ Efficiency,

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{25}{50.6} = 0.494 = 49.4\% \qquad \textbf{Ans.}$$

(*d*) *Maximum efficiency of the machine*

We also know that maximum efficiency of the machine,

$$\eta_{max} = \frac{1}{m \times \text{V.R.}} = \frac{1}{\frac{1}{30} \times 50.6} = 0.593 = 59.3\% \qquad \textbf{Ans.}$$

Screw Jack

It consists of a screw, fitted in a nut, which forms the body of the jack. The principle, on which a screw jack works, is similar to that of an inclined plane. Screw jack is rotated by the application of an effort at the end of the lever, for lifting the load. Now consider a single threaded simple screw jack.

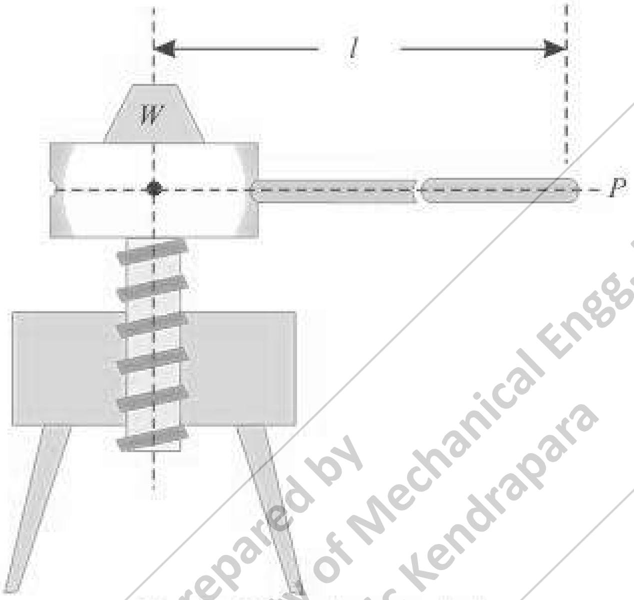


Fig. 11.14. Simple screw jack.

Let l = Length of the handle,

p = Pitch of the screw,

W = Load lifted, and

P = Effort applied to lift the load at the end of the lever.

We know that distance moved by the effort in one revolution of screw,

$$= 2\pi l \qquad \text{...(i)}$$

and distance moved by the load $= p$

Now $$\text{M.A.} = \frac{W}{P} \qquad \text{...as usual}$$

and efficeincy, $$\eta = \frac{\text{M.A.}}{\text{V.R.}} \qquad \text{...as usual}$$

Note: The value of P *i.e.*, the effort applied may also found out by the relation :

$$^{*}P = W \tan(\alpha + \phi)$$

where W = Load lifted

$$\tan \alpha = \frac{p}{\pi d}$$

and $\tan \phi = \mu$ = Coefficient of friction.

Example 11.20. *A screw jack has a thread of 10 mm pitch. What effort applied at the end of a handle 400 mm long will be required to lift a load of 2 kN, if the efficiency at this load is 45%.*

Solution. Given: Pitch of thread (p) = 10 mm; Length of the handle (l) = 400 mm; Load lifted (W) = 2 kN = 2000 N and efficiency (η) = 45% = 0.45.

Let P = Effort required to lift the load.

We know that velocity ratio

$$V.R. = \frac{2\pi l}{p} = \frac{2\pi \times 400}{10} = 251.3$$

and

$$M.A. = \frac{W}{P} = \frac{2000}{P}$$

We also know that efficiency,

$$0.45 = \frac{M.A.}{V.R.} = \frac{\frac{2000}{P}}{251.3} = \frac{7.96}{P}$$

$$P = \frac{7.96}{0.45} = 17.7 \text{ N} \qquad \textbf{Ans.}$$

Gear Trains

Gear

A gear may be defined as a pulley or wheel having projections on its rim known as teeth or cogs. Sometimes, a pulley is casted with teeth on its rim. But, sometimes the teeth are cut on the rim of the pulley.

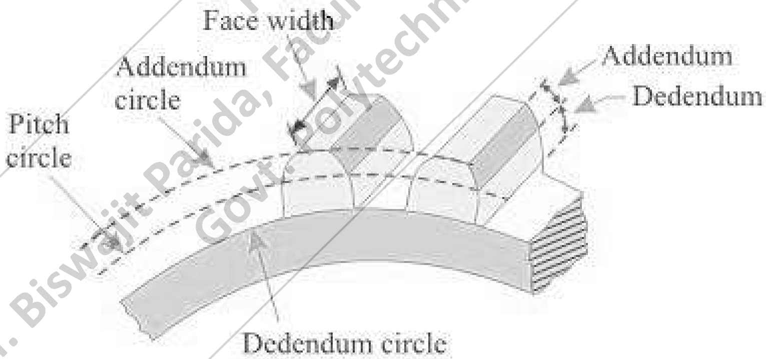
Advantages of a Gear Drive

- It transmits exact velocity ratio.
- It has a high efficiency.
- It has a compact lay out.
- It can transmit a large power.
- It has a reliable service.

Disadvantages of a Gear Drive

- The manufacture of toothed wheels requires special equipment and tools.
- Any error in teeth machinery causes vibrations and noise during operation.
- Any defect in one wheel damages the whole set up.

Important Terms



1) ***Pitch Circle:*** An imaginary circle, which would transmit the same motion as the actual gear, by pure rolling action, is called pitch circle. The diameter of the pitch circle is known as pitch circle diameter.
2) ***Pitch:*** The centre to centre distance between any two teeth, measured along the arc of the pitch circle, is called pitch of the toothed wheel.
   Mathematically, pitch,
   $$p = \pi d/T$$
   where d = Diameter of the pitch circle, and
   T = No. of teeth on the wheels.
3) ***Addendum circle:*** The part of a gear outside the pitch circle is called addendum, and the circle (concentric with the pitch circle) drawn through the top of the teeth is known as addendum circle. The diameter of the addendum circle is known as addendum circle diameter.

4) ***Dedendum circle:*** The part of a gear inside the pitch circle is called dedendum, and the circle (concentric with the pitch circle) drawn through the bottom of the teeth is known as dedendum circle or root circle. The diameter of the dedendum circle is known as dedendum circle diameter.

5) ***Clearance:*** When the two gears mesh together, the addendum of one gear projects inside the dedendum of the other. For smooth working, a small space is left between the addendum circle of one gear and the dedendum circle of the other. This space is known as clearance.

6) ***Depth of the tooth:*** The radial distance between the addendum circle and dedendum circle of a gear is known as depth of the tooth.

7) ***Face of the tooth:*** The surface of a tooth along its width, and above the pitch circle is known as face of the tooth.

8) ***Flank of the tooth:*** The surface of a tooth along its width, and below the pitch circle is known as flank of tooth.

9) ***Face width of the tooth:*** The width of a tooth, measured parallel to its axis, is known as face width of the tooth.

Types of Gears

1) ***External gears:*** Sometimes, the gears of the two shafts mesh externally with each other. Such a type of gear is called external gearing. The larger of these two wheels is called spur wheel and the smaller one is called pinion. In an external gearing, the motion of the two wheels is always unlike.

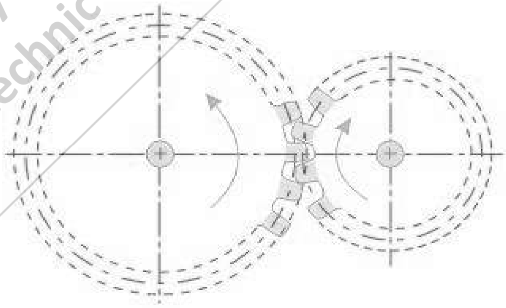


Fig. 34.3. External gearing.

2) ***Internal gears:*** The gear of the two shafts meshes internally with each other. Such a type of gear is called internal gearing. The larger of these two wheels is called annula wheel and the smaller one is called pinion. In an internal gearing, the motion of the two wheels is always like.

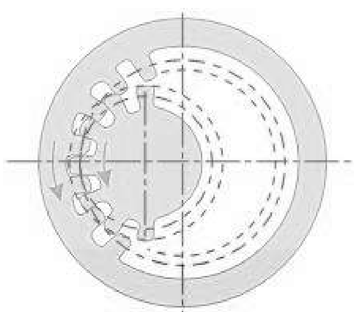


Fig. 34.4. Internal gearing.

3) ***Rack and pinion:*** Sometimes, the gear of a shaft meshes externally and internally with the gears in a straight line. Such a type of gear is called rack and pinion.

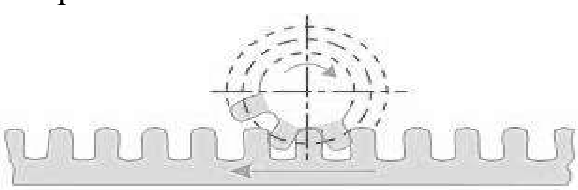


Fig. 34.5. Rack and pinion.

The straight line gear is called rack and the circular wheel is called pinion. A little consideration will show that with the help of a rack and pinion gear, we can convert linear motion into a rotary motion and vice versa.

Simple Gear Drive

A simple gear drive consists of two shafts, containing wheels, with similar teeth, geared together.

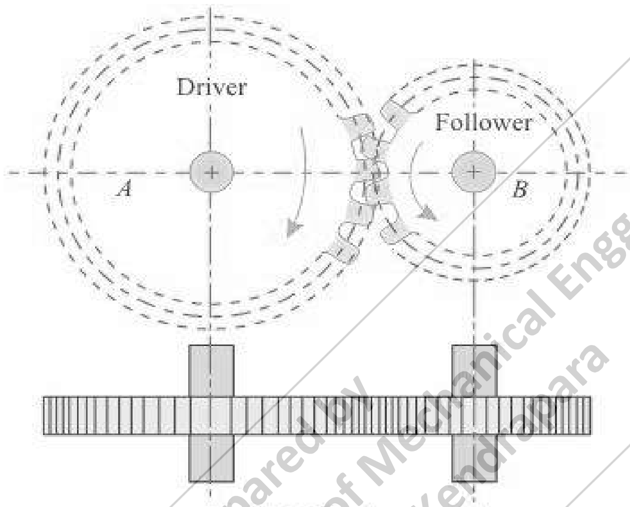


Fig. 34.6. Simple gear drive.

The wheel A is keyed to the rotating shaft, and is known as driver (since it drives the other wheel). The wheel B is keyed to shaft, intended to be rotated and is known as follower or driven (since it is driven by the wheel A). When the driver rotates, its teeth drive the teeth of the follower, which rotate it in the opposite direction of its motion.

Velocity Ratio

It is the ratio between the velocities of the driver and the follower or driven. Now consider a simple gear drive.

Let N_1 = Speed of the driver,

T_1 = No. of teeth on the driver,

d_1 = Diameter of the pitch circle of the driver,

N_2, T_2, d_2 = Corresponding values for the follower, and

p = Pitch of the two wheels.

We know that the pitch of the driver

$$p = (\pi d_1)/T_1 \quad ...(i)$$

Similarly, pitch of the follower

$$p = (\pi d_2)/T_2 \quad ...(ii)$$

Since the pitch of both the wheels is the same, therefore equating *(i)* and *(ii)*,

$$(\pi d_1)/T_1 = (\pi d_2)/T_2$$

$$(d_1/d_2) = (T_1/T_2)$$

∴ Velocity ratio $(N_2/N_1) = (d_1/d_2) = (T_1/T_2)$

Power Transmitted

Consider a simple gear drive transmitting power, from one shaft to another.

Let F = Tangential force exerted by the driver (also called pressure between the teeth) and

v = Peripheral velocity of the driver, at the pitch point,

∴ Power transmitted or work done

$$P = Force \times Distance = Fv$$

Example 34.1. *In a spur gear arrangement, the driver has 100 teeth of 40 mm pitch. Find the power, if it can transmit a tangential force of 100 N on the follower. Take speed of driver as 225 r.p.m.*

Solution. Given: No. of teeth on driver (T) = 100; Pitch of the two wheels (p) = 40 mm; Tangential force exerted by the driver (F) = 100 N and speed of the driver (N) = 225 r.p.m.

We know that circumference of the pitch circle

$$= 100 \times 40 = 4000 \text{ mm} = 4 \text{ m}$$

∴ Velocity of driver at the pitch point

$$v = \frac{4 \times 225}{60} = 15 \text{ m/s}$$

and power transmitted by the gear,

$$P = F \times v = 100 \times 15 = 1500 \text{ N-m/s}$$
$$= 1500 \text{ W} = 1{\cdot}5 \text{ kW} \quad \textbf{Ans.}$$

Gear Train

Sometimes, two or more gears are made to mesh with each other, so as to operate as a single system, to transmit power from one shaft to another. Such a combination is called gear train or train of wheels. Following are the two types of train of wheels depending upon the arrangement of wheels:

1) Simple Gear Train

Sometimes the distance between the two wheels is great. The motion from one wheel to another, in such a case, may be transmitted by either of the following two methods: a) By providing a large sized wheel, or b) By providing intermediate wheels.

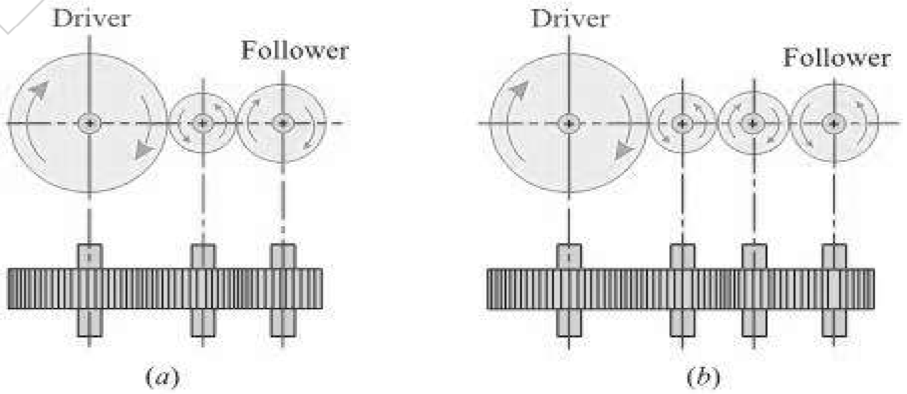


Fig. 34.7. Simple train of wheels.

It may be noted that when the number of intermediate wheels is odd, the motion of both the wheels (i.e., driver and follower) is like. But, if the number of intermediate wheels is even, the motion of the follower is the opposite direction of the driver.

Now consider a simple train of wheels with one intermediate wheel.

Let N_1 = Speed of the driver

T_1 = No. of teeth on the driver,

N_2, T_2 = Corresponding values for the intermediate wheel, and N_3, T_3 = Corresponding values for the follower.

Since the driver gears with the intermediate wheel, therefore

$$(N_2/N_1) = (T_1/T_2) \qquad ...(i)$$

Similarly, as the intermediate wheel gears with the follower, therefore

$$(N_3/N_2) = (T_2/T_3) \qquad ...(ii)$$

Multiplying equation *(ii)* by *(i)*,

$$(N_3/N_2) \times (N_2/N_1) = (T_2/T_3) \times (T_1/T_2)$$

$$(N_3/N_1) = (T_1/T_3)$$

$$\therefore \frac{\text{Speed of the follower}}{\text{Speed of the driver}} = \frac{\text{No. of teeth on the driver}}{\text{No. of teeth on the follower}}$$

Similarly, it can be proved that the above equation also holds good, even if there are any number of intermediate wheels. It is thus obvious, that the velocity ratio, in a simple train of wheels, is independent of the intermediate wheels. These intermediate wheels are also called idle wheels, as they do not affect the velocity ratio of the system.

2) Compound Gear Train

Whenever the distance between the driver and follower has to be bridged over by intermediate wheels and at the same time a great (or much less) velocity ratio is required then the advantage of intermediate wheels in intensified by providing compound wheels on intermediate shafts.

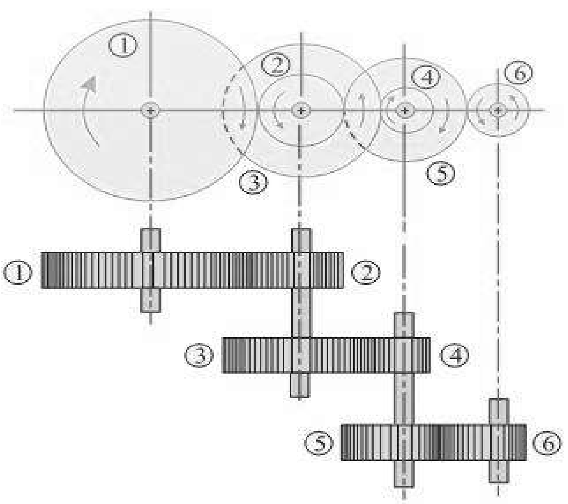


Fig. 34.8. Compound train of wheels.

In this case, each intermediate shaft has two wheels rigidly fixed to it, so that they may have the same speed. One of these two wheels gears with the driver and the other with the follower attached to the next shaft.

Let N_1 = Speed of the driver 1,

T_1 = No. of teeth on the driver 1,

Similarly $N_2, N_3, \ldots N_6$ = Speed of the respective wheels,

$T_2, T_3, \ldots T_6$ = No. of teeth on the respective wheels.

Since the wheel *1* gears with the wheel *2*, therefore

$$(N_2/N_1) = (T_1/T_2) \quad \ldots(i)$$

Similarly $$(N_4/N_3) = (T_3/T_4) \quad \ldots(ii)$$

And $$(N_6/N_5) = (T_5/T_6) \quad \ldots(iii)$$

Multiplying equations *(i)*, *(ii)* and *(iii)*,

$$(N_2/N_1) \times (N_4/N_3) \times (N_6/N_5) = (T_1/T_2) \times (T_3/T_4) \times (T_5/T_6)$$

$$\therefore \quad (N_6/N_1) = (T_1 \times T_3 \times T_5) / (T_2 \times T_4 \times T_6)$$

$$= \frac{\text{Product of the teeth on the drivers}}{\text{Product of the teeth on the followers}}$$

Example 34.2. *The gearing of a machine tools is shown in Fig. 34.9.*

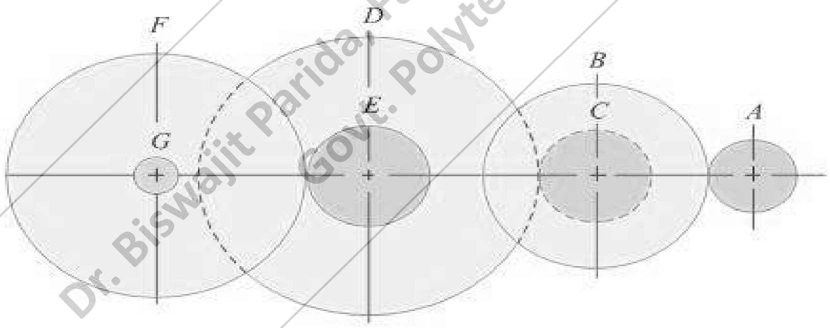


Fig. 34.9.

The motor shaft is connected to A and rotates at 975 r.p.m. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft G. What is the speed of F ? The number of teeth on each wheel is as given below :

Gear	*A*	*B*	*C*	*D*	*E*	*F*
No. of teeth	*20*	*50*	*25*	*75*	*26*	*65*

Solution. Given: Speed of the gear wheel A (N_A) = 975 r.p.m.; No. of teeth on wheel A (T_A) = 20; No. of teeth on wheel B (T_B) = 50; No. of teeth on wheel C (T_C) = 25; No. of teeth on wheel D (T_D) = 75; No. of teeth on wheel E (T_E) = 26 and no. of teeth on wheel F (T_F) = 65.

Let N_F = Speed of the shaft F.

We know that $$\frac{N_F}{N_A} = \frac{T_A \times T_C \times T_E}{T_B \times T_D \times T_F}$$

$$\therefore \quad \frac{N_F}{975} = \frac{20 \times 25 \times 26}{50 \times 75 \times 65} = \frac{4}{75}$$

or $$N_F = 975 \times \frac{4}{75} = 52 \text{ r.p.m.}$$ **Ans.**

Example 34.3. *A handle H drives a pinion A, which drives a drum E through gear wheels B, C and D as shown in Fig. 34.10.*

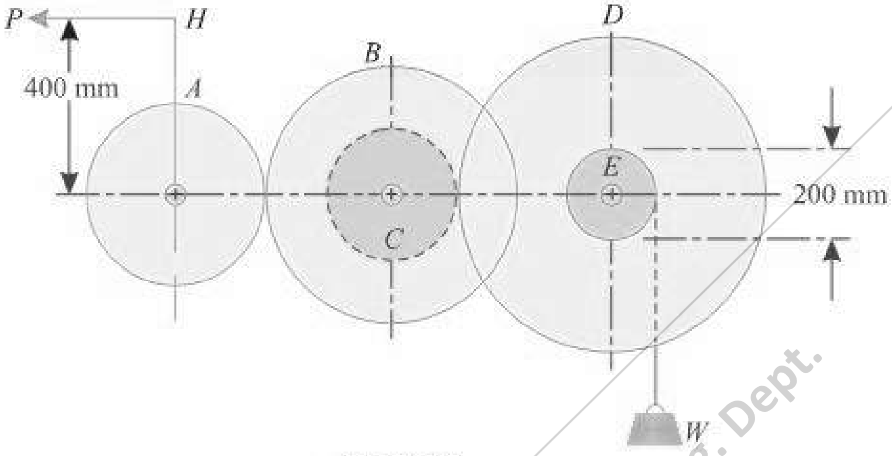


Fig. 34.10.

The length of handle is 400 mm, and diameter of the drum is 200 mm. The wheel A has 20 teeth, which gears with wheel B of 80 teeth. The wheel C has 20 teeth which gears with wheel D of 100 teeth.

Find the load (W) that can be raised by the drum, if an effort of 10 N is applied at the end of the handle. Take efficiency of the system as 60%.

Solution. Given: Length of the handle (l) = 400 mm = 0·4 m; Diameter of the drum (d) = 200 mm = 0·2 m or radius (r) = 0·1 m; No. of teeth on wheel A (T_A) = 20; No. of teeth on wheel B (T_B) = 80; No. of teeth on wheel C (T_C) = 20; No. of teeth on wheel D (T_D) = 100; Effort applied (P) = 10 N and efficiency of the system (η) = 60% = 0·6.

Let W = Load that can be raised by the drum.

A little consideration will show, that this example is exactly like that of a double purchase crap winch. We know that velocity ratio of the system,

$$\text{V. R.} = \frac{l}{r}\left(\frac{T_B \times T_D}{T_A \times T_C}\right) = \frac{0{\cdot}4}{0{\cdot}1}\left(\frac{80 \times 100}{20 \times 20}\right) = 80$$

$$\text{M.A.} = \frac{W}{P} = \frac{W}{10}$$

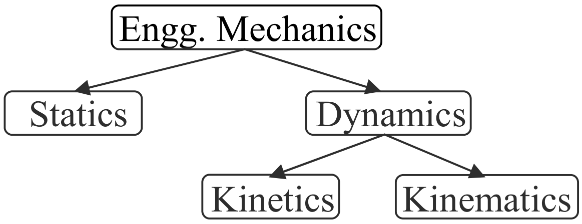
and efficiency,

$$0{\cdot}6 = \frac{\text{M.A.}}{\text{V.R.}} = \frac{\frac{W}{10}}{80} = \frac{W}{800}$$

$$\therefore \quad W = 0{\cdot}6 \times 800 = 480 \text{ N} \qquad \textbf{Ans.}$$

Chapter-06: Dynamics

6.1 Dynamics



Dynamics: It deals with the forces and their effects, while acting upon the bodies in motion.

Kinetics: It deals with the bodies in motion due to the application of forces.

Kinematics: It deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

Important Terms

Speed: The speed of a body may be defined as its rate of change of displacement with respect to its surroundings. The speed of a body is irrespective of its direction and is, thus, a scalar quantity.

Velocity: The velocity of a body may be defined as its rate of change of displacement, with respect to its surroundings, in a particular direction. As the velocity is always expressed in particular direction, therefore it is a vector quantity.

Acceleration: The acceleration of a body may be defined as the rate of change of its velocity. It is said to be positive, when the velocity of a body increases with time, and negative when the velocity decreases with time. The negative acceleration is also called retardation.

Uniform Acceleration: If a body moves in such a way that its velocity changes in equal magnitudes in equal intervals of time, it is said to be moving with a uniform acceleration.

Variable Acceleration: If a body moves in such a way, that its velocity changes in unequal magnitudes in equal intervals of time, it is said to be moving with a variable acceleration.

Distance Traversed: It is the total distance moved by a body. Mathematically, if body is moving with a uniform velocity (v), then in (t) seconds, the distance traversed

$$s = vt$$

Mass: It is the matter contained in a body. The units of mass are kilogram, tonne etc.

Weight: It is the force, by which the body is attracted towards the centre of the earth. The units of weight are the same as those of force i.e. N, kN etc.

Momentum: It is the quantity of motion possessed by a body. It is expressed mathematically as

$$\text{Momentum} = \text{Mass} \times \text{Velocity}$$

The units of momentum depend upon the units of mass and velocity. In S.I. units, the mass is measured in kg, and velocity in m/s, therefore the unit of momentum will be kg-m/s.

Force: It is a very important factor in the field of dynamics also, and may be defined as any cause which produces or tends to produce, stops or tends to stop motion. The units of force, like those of weight, are N, kN etc.

Inertia: It is an inherent property of a body, which offers resistance to the change of its state of rest or uniform motion.

Equations of Motion

It states "Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force." It is also called the law of inertia, and consists of the

Newton's Laws of Motion

Newton's First Law of Motion

It states "Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force." It is also called the law of inertia, and consists of the following two parts:

1. A body at rest continues in the same state, unless acted upon by some external force. It appears to be self-evident, as a train at rest on a level track will not move unless pulled by an engine. Similarly, a book lying on a table remains at rest, unless it is lifted or pushed.

2. A body moving with a uniform velocity continues its state of uniform motion in a straight line, unless it is compelled by some external force to change its state. It cannot be exemplified because it is, practically, impossible to get rid of the forces acting on a body.

Newton's Second Law of Motion

It states, "The rate of change of momentum is directly proportional to the impressed force and takes place, in the same direction in which the force acts." This law enables us to measure a force, and establishes the fundamental equation of dynamics. Now consider a body moving in a straight line. Let its velocity be changed while moving.

Let m = Mass of a body,

u = Initial velocity of the body,

v = Final velocity of the body,

a = Constant acceleration,

t = Time, in seconds required to change the velocity from u to v, and

F = Force required to change velocity from u to v in t seconds.

∴ Initial momentum = mu

and final momentum = mv

∴ Rate of change of momentum

$$= \frac{mv - mu}{t} = \frac{m(v-u)}{t} = ma \quad \ldots \left[\because \frac{v-u}{t} = a\right]$$

According to Newton's Second Law of Motion, the rate of change of momentum is directly proportional to the impressed force.

∴ $F \propto ma = kma$

Where k is a constant of proportionality. For the sake of convenience, the unit of force adopted is such that it produces unit acceleration to a unit mass.

∴ $F = ma$ = Mass × Acceleration.

In S.I. system of units, the unit of force is called Newton briefly written as N. A Newton may be defined as the force while acting upon a mass of 1 kg, produces an acceleration of 1 m/s^2 in the direction of which it acts. It is also called the Law of dynamics and consists of the following two parts:

1. A body can posses acceleration only when some force is applied on it. Or in other words, if no force is applied on the body, then there will be no acceleration, and the body will continue to move with the existing uniform velocity.

2. The force applied on a body is proportional to the product of the mass of the body and the acceleration produced in it.

It will be interesting to know that first part of the above law appears to be an extension of the First Law of Motion. However, the second part is independent of the First Law of Motion.

Newton's Third Law of Motion

It states, "If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant

A simple machine may be defined as a device, which enables us to do some useful work at some point or to overcome some resistance, when an effort or force is applied to it, at some other convenient point.

D'Alembert's Principle

It states, "If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction and the line of action may be found out by the methods of graphic statics."

We have already discussed in art. 24.6, that force acting on a body.

$$P = ma \qquad \text{...(i)}$$

where m = mass of the body, and

a = Acceleration of the body.

The equation (i) may also be written as :

$$P - ma = 0 \qquad \text{...(ii)}$$

It may be noted that equation (i) is the equation of dynamics whereas the equation (ii) is the equation of statics. The equation (ii) is also known as the equation of dynamic equilibrium under the action of the real force P. This principle is known as D' Alembert's principle.

Example 24.17. *Two bodies A and B of mass 80 kg and 20 kg are connected by a thread and move along a rough horizontal plane under the action of a force 400 N applied to the first body of mass 80 kg as shown in Fig. 24.2.*

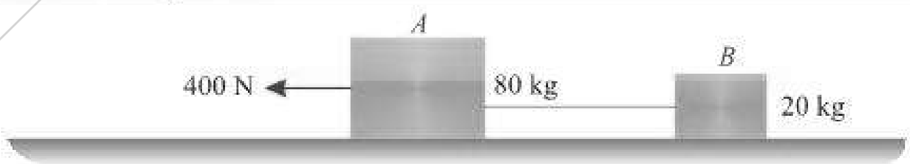


Fig. 24.2.

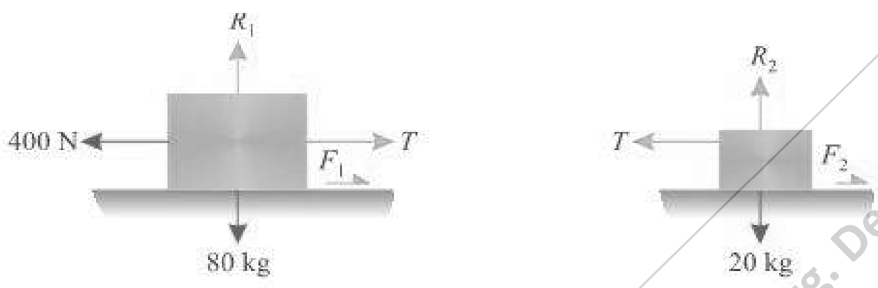
The coefficient of friction between the sliding surfaces of the bodies and the plane is 0.3. Determine the acceleration of the two bodies and the tension in the thread, using D' Alembert's principle.

Solution. Given : Mass of body A (m_1) = 80 kg ; Mass of the body B (m_2) = 20 kg; Force applied on first body (P) = 400 N and coefficient of friction (μ) = 0.3

Acceleration of the two bodies

Let a = Acceleration of the bodies, and

T = Tension in the thread.



(*a*) *Body A* (*b*) *Body B*

Fig. 24.3.

First of all, consider the body A. The forces acting on it are :

1. 400 N force (acting towards left)
2. Mass of the body = 80 kg (acting downwards)
3. Reaction $R_1 = 80 \times 9.8 = 784$ N (acting upwards)
4. Force of friction, $F_1 = \mu R_1 = 0.3 \times 784 = 235.2$ N (acting towards right)
5. Tension in the thread = T (acting towards right).

∴ Resultant horizontal force,

$$P_1 = 400 - T - F_1 = 400 - T - 235.2$$
$$= 164.8 - T \text{ (acting towards left)}$$

We know that force causing acceleration to the body A

$$= m_1 a = 80\ a$$

and according to D' Alembert's principle ($P_1 - m_1 a = 0$)

$$164.8 - T - 80\ a = 0$$

or $$T = 164.8 - 80a \qquad ...(i)$$

Now consider the body B. The forces acting on it are :

1. Tension in the thread = T (acting towards left)
2. Mass of the body = 20 kg (acting downwards)
3. Reaction $R_2 = 20 \times 9.8 = 196$ N (acting upwards)
4. Force of friction, $F_2 = \mu R_2 = 0.3 \times 196 = 58.8$ N (acting towards right)

∴ Resulting horizontal force,

$$P_2 = T - F_2 = T - 58.8$$

We know that force causing acceleration to the body B

$$= m_2 a = 20\ a$$

and according to *D'* Alembert's principle ($P_2 - m_2 a = 0$)

$$(T - 58.8) - 20\,a = 0$$

or $$T = 58.8 + 20\,a \quad ...(ii)$$

Now equating the two values of T from equation (i) and (ii),

$$164.8 - 80\,a = 58.8 + 20\,a$$

$$100\,a = 106$$

or $$a = \frac{106}{100} = 1.06 \text{ m/s}^2 \quad \textbf{Ans.}$$

Tension in the thread

Substituting the value of a in equation (ii),

$$T = 58.8 + (20 \times 1.06) = 80 \text{ N} \quad \textbf{Ans.}$$

Law of a Machine

may be defined as relationship between the effort applied and the load lifted. Thus for any machine, if we record the various efforts required to raise the corresponding loads, and plot a graph between effort and load, we shall get a straight line.

- The centre of area of plane figures (like triangle, quadrilateral, circle etc.) is known as centroid.
- The method of finding out the centroid of a figure is the same as that of finding out the centre of gravity of a body.

Moment Of Inertia of a Rectangular Section

Consider

4.2 Moment of Inertia

- Whenever one of the body moves or tends to move tangentially with respect to the surface, on which it rests, the interlocking property of the projecting particles opposes the motion.
- This opposing force, which acts in the opposite direction of the movement of the body, is called force of friction or simply friction.

It is of the following two types:

1. Static friction. 2. Dynamic friction.

Polygon Law of Forces

c) ***Static Friction:*** It is the friction experienced by a body when it is at rest. Or in other words, it is the friction when the body tends to move..

d) ***Dynamic Friction:*** It is the friction experienced by a body when it is in motion. It is also called kinetic friction.